Fault tolerance of cooperative interception using multiple flight vehicles

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Abstract

Cooperative interception of a moving target by multiple vehicles is studied. The main contributions of research work presented in this paper include: (1) cooperative interception is achieved for multiple vehicles to reach the target simultaneously at a finite time, by proposing and solving for a novel finite-time consensus problem and (2) in addition, the cooperative interception is investigated with tolerance of actuator or network failures, where novel fault tolerant consensus protocols are proposed to address actuator failures (or loss of effectiveness) and network failures, respectively. The maximum fault tolerance against network failures can be estimated. Simulations of a three-against-one interception case are presented to demonstrate the effectiveness of the proposed design approaches.

1. Introduction

Multi-vehicle systems have great potential in various applications such as tracking and interception, search and rescue, surveillance and cooperative monitoring [1–4]. In interception missions, a strategy of multi-vehicle systems against one target (many-to-one) becomes a more favorable strategy over the traditional one-by-one engagement approach, as the latter may have a failed interception. Specifically in the many-to-one mission, it is important that multiple flight vehicles reach the target simultaneously [5–7].

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Some many-to-one engagements especially for a static or low-speed target are studied in [5,7,8]. Jeon et al. [5] proposed an impact-time-control guidance (ITCG) law, whose impact time must be calculated in priori and set into all vehicles off-line. Since the vehicles do not exchange information during the guidance, the ITCG-based approach is a static guidance strategy. By taking advantage of the communication among the vehicles, a cooperative proportional navigation for many-to-one engagement was proposed in [8]. However, the proposed centralized control law in each vehicle requires real-time update of global information of the entire system. Zhao and Zhou [7] extended the work of [8] to a two-level hierarchical architecture for the cooperative guidance, where the centralized and decentralized frames are constructed based on the impact-time. However, these studies only aimed at a static or low-speed target. To the authors' best knowledge, the study of a single moving target intercepted by multiple flight vehicles is rare. A cooperative interception problem, defined as that multiple flight vehicles intercept a single moving target simultaneously, is desirable.

An implicit constraint in cooperative interception is that the engagement must be completed in finite time. This imposes a finite-time cooperation problem. There are some studies on the finite-time stability and cooperation. Bhat et al. addressed the sufficient condition of finite-time stability in [9,10] and proposed a design method for state feedback controller. Furthermore, an output feedback controller was proposed in [11] by Hong et al. to stabilize the nonlinear systems in finite time. With the development of cooperative control, the finite-time consensus of multi-agent systems is attracting more and more attentions from researchers such as Cortes [12], Xiao et al. [13,14] and Wang et al. [15]. Two finite-time consensus protocols for continuous-time systems are proposed in [12], both of which are discontinuous. Xiao and Wang developed a finite-time consensus protocol in [13,14] that incorporates continuous state feedback. However, most studies on finite-time consensus protocols ignore the possible failures such as actuator failures and network failures. These failures deteriorate the consensus performance that will jeopardize the cooperative interception mission. Although Wang and Xiao considered the switching communication topology as the network failures in [15], the maximum tolerance of network failures for a time-decay system is hard to estimate.

Motivated by the limitations discussed above, we are proposing a novel formulation for cooperative interception, with emphasis on fault tolerance. This formulation is based on the finite-time consensus. Specifically, the relative kinematics between individual flight vehicle and the target is constructed in the line-of-sight (LOS) frame. The guidance law design is then divided into two stages. In the first stage, the command acceleration is designed to guarantee that the relative distance converges to zero, i.e., the individual interception is guaranteed. The second stage is to ensure that all the vehicles reach the moving target simultaneously. By introducing a novel dynamic consensus variable, denoted by time-to-go, to represent the time required for interception, the second stage is formulated as a dynamic finite-time consensus problem. Two novel consensus protocols are proposed with the special emphasis on their fault tolerances to possible failures such as actuator failures and network failures.

The main contributions are summarized as follows:

1. For the first protocol, we show that for the vehicle endowed by head or tail actuators, it can tolerate either one of the failures. In other words, the cooperative interception can still be accomplished if some actuators become partially effective. In addition, we also show that if some actuators have partial loss of effectiveness (i.e., the operation range is limited), the cooperative interception is still guaranteed.
2. For the second protocol, we show that a finite-time consensus protocol can guarantee cooperative interception if the network failures happen. In addition, to define the unconnected time of topology as a tolerance degree, the maximum fault tolerance against network failures can be estimated.

3. Different from [13,14] for static variable consensus, this work combines the flight dynamics and dynamic consensus. In addition, this work puts more emphasis on the fault tolerance. Specially, through the introduction of sign+, sign−, sat+ and sat− in the controller structure, the fault tolerance is guaranteed.

4. Finally, compared to [16] for one-by-one guidance law, our work can handle many-to-one engagement.

The rest of paper is organized as follows: Some results on the finite-time consensus are given in Section 2. Section 3 describes the formulation of cooperative interception with single moving target. In Section 4, a finite-time consensus protocol is proposed to ensure that multiple flight vehicles intercept the target simultaneously when there exist actuator failures. The other finite-time consensus protocol is proposed whose maximum tolerance against network failures is estimated in Section 5. Simulation examples and analysis are shown in Section 6. Some current results and future directions are concluded in Section 7.

2. Finite-time consensus

Since our problem formulation of cooperative interception is based on finite-time consensus, we will give some preliminary results on finite-time consensus here. In particular, we will give some novel finite-time consensus protocols, which underlay the cooperative interception formulated later, providing some fault tolerances.

For an \( n \)-agent system, the dynamics of agent \( i \) can be described as

\[
\dot{x}_i = u_i
\]

where \( x_i \) is the state of agent \( i \) and \( u_i \) is called protocol, which is the function of the states from its own and neighbors.

The network formed by multi-agent systems can be represented by graphs. A weighted undirected graph \( G(A) = (\mathcal{V}, \mathcal{E}, A) \) is used to represent the communication topology, where \( \mathcal{V} \) represents the set of vertices and \( \mathcal{E} \) represents the edge set. Node \( v_i \) corresponds to agent \( i \). An edge of \( G(A) \) is denoted by \( (v_i, v_j) \), which is an unordered pair of vertices. \( (v_i, v_j) \in \mathcal{E} \iff a_{ij} > 0 \iff \) agents \( i \) and \( j \) can exchange the information with each other, namely, they are adjacent. Since the graph is undirected, \( a_{ij} = a_{ji} \). Moreover, we assume that \( a_{ii} = 0 \) for all \( i = 1,2,\ldots,n \). Matrix \( A \) is called the weight matrix and \( a_{ij} \) is the weight of edge \( (v_i, v_j) \). In consistence with the definition of agents’ neighbors, the set of neighbors of vertex \( v_i \) is denoted by \( \mathcal{N}_i = \{v_j \in \mathcal{V} | (v_j, v_i) \in \mathcal{E}\} \). A path in a graph from \( v_i \) to \( v_j \) is a sequence of distinct vertices starting with \( v_i \) and ending with \( v_j \) such that consecutive vertices are adjacent. A graph is connected if there is a path between any two vertices of the graph.

**Definition 1.** Given protocol \( u_i, i = 1,2,\ldots,n \), \( u_i \) or this multi-agent system is said to solve an agreement problem if for any given initial states and any \( j, k = 1,2,\ldots,n \), \( |x_j(t) - x_k(t)| \to 0 \), as \( t \to \infty \).

**Definition 2.** Given protocol \( u_i, i = 1,2,\ldots,n \), \( u_i \) or this multi-agent system is said to solve a finite-time agreement problem if for any initial states, there exist a time \( t^* \) and a real number \( \kappa \) such that \( x_j(t) = \kappa \) for \( t \geq t^* \).
Definition 3. Given protocol \( u_i, i = 1, 2, \ldots, n \), or this multi-agent system is said to solve an average-consensus problem if the final agreement state is the average of the initial states, namely, 
\[
x_j(t) \to \frac{1}{n} \sum_{k=1}^{n} x_k(0) \quad \text{for all } j = 1, 2, \ldots, n \text{ as } t \to \infty.
\]

For the graph topology \( A = [a_{ij}] \in \mathbb{R}^{n \times n} \), we introduce a graph Laplacian \( L(A) \in \mathbb{R}^{n \times n} \), whose component is defined as [17]
\[
l_{ij} = \begin{cases} 
\sum_{k=1, k \neq i}^{n} a_{ik} & j = i \\
-a_{ij} & j \neq i.
\end{cases}
\]

A graph Laplacian \( L(A) \) has the following properties:

1. 0 is an eigenvalue of \( L(A) \) and \( \mathbf{1} \) is the associated eigenvector.
2. \( X^T L(A) X = \frac{1}{2} \sum_{i,j=1}^{n} a_{ij} (x_j - x_i)^2 \) and the semi-positive definiteness of \( L(A) \) implies that all eigenvalues of \( L(A) \) are real and not less than zero.
3. If \( G(A) \) is connected, the second smallest eigenvalue of \( L(A) \), which is denoted by \( \lambda_2(L(A)) \) and called the algebraic connectivity of \( G(A) \), is larger than zero.
4. The algebraic connectivity of \( G(A) \) is equal to \( \min_{X \neq 0} X^T L(A) X / X^T X \) and therefore, if \( 1^T X = 0 \), \( X^T L(A) X \geq \lambda_2(L(A)) X^T X \).

The standard sign function \( \text{sign}(\cdot) \) is generally defined as
\[
\text{sign}(z) = \begin{cases} 
1 & z > 0 \\
0 & z = 0 \\
-1 & z < 0.
\end{cases}
\]

In this paper, we will introduce several new notations, for the convenience of delivering the new protocols later. Specifically, the plus-sign function \( \text{sign}^+(\cdot) \) and minus-sign function \( \text{sign}^-(\cdot) \) are defined as
\[
\text{sign}^+(z) = \begin{cases} 
1 & z > 0 \\
0 & z \leq 0 \\
1 & z < 0.
\end{cases}
\]
\[
\text{sign}^-(z) = \begin{cases} 
0 & z \geq 0 \\
1 & z < 0.
\end{cases}
\]

The semi-saturation functions \( \text{sat}^+(\cdot) \) and \( \text{sat}^-(\cdot) \) are defined as
\[
\text{sat}^+(z) = \begin{cases} 
z_{\text{max}} & z > z_{\text{max}} \geq 0 \\
z & z \leq z_{\text{max}} \\
z_{\text{max}} & z < z_{\text{max}} \leq 0.
\end{cases}
\]
\[
\text{sat}^-(z) = \begin{cases} 
z & z \geq z_{\text{min}} \\
z_{\text{min}} & z < z_{\text{min}} \leq 0.
\end{cases}
\]

Remark 1. The new defined sign and sat functions can be thought of as an extension of the standard sign function. They will be used to construct new protocols with given fault tolerance.
The following lemma is widely used to show finite-time stability and consensus.

**Lemma 1** (Bhat and Bernstein [10]). Suppose the function \( V(t) : [0, \infty) \rightarrow [0, \infty) \) is differentiable (the derivative of \( V(t) \) at 0 is in fact its right derivative) and
\[
\frac{dV(t)}{dt} \leq -KV(t)^\beta
\]
where \( K > 0 \) and \( 0 < \beta < 1 \). Then \( V(t) \) will reach zero at finite time \( t^* \leq V(0)^{1-\beta}/K(1-\beta) \) and \( V(t) = 0 \) for all \( t \geq t^* \).

**Lemma 2.** If \( \sum_{i=1}^{n} z_i = 0 \), then
\[
\sum_{i=1}^{n} \left( \text{sign}^+ z_i \cdot z_i \right)^2 \geq \frac{1}{n} \sum_{i=1}^{n} z_i^2
\]
(2)
\[
\sum_{i=1}^{n} \left( \text{sign}^- z_i \cdot z_i \right)^2 \geq \frac{1}{n} \sum_{i=1}^{n} z_i^2
\]
(3)

**Proof.** We define the sets \( J^+ \) and \( J^- \) which consist of all the positive numbers and all the negative numbers in \( \{z_1, z_2, \ldots, z_n\} \), respectively, hence we have
\[
\sum_{i=1}^{n} z_i^2 = \sum_{z_{pos} \in J^+} z_{pos}^2 + \sum_{z_{neg} \in J^-} z_{neg}^2.
\]
By \( \sum_{z_{pos} \in J^+} z_{pos}^2 \leq (\sum_{z_{pos} \in J^+} z_{pos})^2 \) and \( \sum_{z_{neg} \in J^-} z_{neg}^2 \leq (\sum_{z_{neg} \in J^-} z_{neg})^2 \), we have
\[
\sum_{i=1}^{n} z_i^2 \leq \sum_{z_{pos} \in J^+} z_{pos}^2 + \left( \sum_{z_{pos} \in J^+} z_{pos} \right)^2
\]
(4)
\[
\sum_{i=1}^{n} z_i^2 \leq \sum_{z_{neg} \in J^-} z_{neg}^2 + \left( \sum_{z_{neg} \in J^-} z_{neg} \right)^2.
\]
(5)
By the condition of \( \sum_{i=1}^{n} z_i = 0 \)
\[
\left( \sum_{z_{pos} \in J^+} z_{pos} \right)^2 = \left( \sum_{z_{neg} \in J^-} z_{neg} \right)^2.
\]
Inequalities (4) and (5) can be rewritten as
\[
\sum_{i=1}^{n} z_i^2 \leq \sum_{z_{pos} \in J^+} z_{pos}^2 + \left( \sum_{z_{pos} \in J^+} z_{pos} \right)^2
\]
(6)
\[
\sum_{i=1}^{n} z_i^2 \leq \sum_{z_{neg} \in J^-} z_{neg}^2 + \left( \sum_{z_{neg} \in J^-} z_{neg} \right)^2.
\]
(7)
Referring to Cauchy–Schwarz inequality for the terms in the above equation, we have
\[
\left( \sum_{z_{pos} \in J^+} z_{pos} \right)^2 \leq n^+ \sum_{z_{pos} \in J^+} z_{pos}^2
\]
(8)
\[
\left( \sum_{z_{\text{neg}} \in J^-} z_{\text{neg}} \right)^2 \leq n^- \sum_{z_{\text{neg}} \in J^-} z_{\text{neg}}^2
\]  

(9)

where \(n^+\) and \(n^-\) denote the numbers of elements in \(J^+\) and \(J^-\), respectively.

According to inequalities (8) and (9), we have

\[
\sum_{i=1}^n z_i^2 \leq (n^+ + 1) \sum_{z_{\text{pos}} \in J^+} z_{\text{pos}}^2
\]

(10)

\[
\sum_{i=1}^n z_i^2 \leq (n^- + 1) \sum_{z_{\text{neg}} \in J^-} z_{\text{neg}}^2.
\]

(11)

Since \(\sum_{i=1}^n z_i = 0\), we have \(n^+ \leq n-1\) and \(n^- \leq n-1\). By the truth of \(\sum_{i=1}^n (\text{sign}^+ z_i \cdot z_i)^2 = \sum_{z_{\text{pos}} \in J^+} z_{\text{pos}}^2\) and \(\sum_{i=1}^n (\text{sign}^- z_i \cdot z_i)^2 = \sum_{z_{\text{neg}} \in J^-} z_{\text{neg}}^2\), we have

\[
\sum_{i=1}^n (\text{sign}^+ z_i \cdot z_i)^2 \geq \frac{1}{n} \sum_{i=1}^n z_i^2 \sum_{i=1}^n (\text{sign}^- z_i \cdot z_i)^2 \geq \frac{1}{n} \sum_{i=1}^n z_i^2.
\]

\(\square\)

With the preparation of Lemmas 1 and 2, we are ready to propose a new consensus protocol, as stated in Theorem 1.

**Theorem 1.** If communication topology \(G(A)\) is connected, then either one of the following protocols

\[
u_i = \text{sign}^+ \left( \sum_{v_j \in N_i} a_{ij} (x_j - x_i) \right) \sum_{v_j \in N_i} a_{ij} (x_j - x_i)^\alpha
\]

(12)

\[
u_i = \text{sign}^- \left( \sum_{v_j \in N_i} a_{ij} (x_j - x_i) \right) \sum_{v_j \in N_i} a_{ij} (x_j - x_i)^\alpha
\]

(13)

(0 < \alpha < 1) solves a finite-time agreement problem.

**Proof.** We only prove that protocol (12) solves a finite-time agreement problem, and the proof of protocol (13) follows a similar way. Take semi-positive definite function

\[
V_1(t) = \frac{1}{4} \sum_{i,j=1}^n a_{ij} (x_j - x_i)^2 = \frac{1}{2} x^T L(A)x
\]

(14)

which will be proven to be a valid Lyapunov function for agreement analysis. Since \(G(A)\) is connected, \(V_1(t) = 0\) implies that \(x_j = x_k\) for any \(j, k = 1, 2, \ldots, n\). The symmetry of \(A\) gives that

\[
\frac{\partial V_1(t)}{\partial x_i} = - \sum_{v_j \in N_i} a_{ij} (x_j - x_i).
\]

According to the definition of \(\text{sign}^+(\cdot)\), we have

\[
\text{sign}^+ \left( \sum_{v_j \in N_i} a_{ij} (x_j - x_i) \right) = \begin{cases} 
1 & \sum_{v_j \in N_i} a_{ij} (x_j - x_i) > 0 \\
0 & \sum_{v_j \in N_i} a_{ij} (x_j - x_i) \leq 0 
\end{cases}
\]
Hence, we have
\[
\frac{dV_1(t)}{dt} = \sum_{i=1}^{n} \frac{dV_1(t)}{dx_i} \cdot \dot{x}_i = -\sum_{i=1}^{n} \left( \text{sign}^+ \left( \sum_{v_j \in \mathcal{N}_i} a_{ij}(x_j-x_i) \right) \sum_{v_j \in \mathcal{N}_i} a_{ij}(x_j-x_i)^{1+\alpha} \right) \\
= -\sum_{i=1}^{n} \left( \left( \text{sign}^+ \left( \sum_{v_j \in \mathcal{N}_i} a_{ij}(x_j-x_i) \right) \sum_{v_j \in \mathcal{N}_i} a_{ij}(x_j-x_i) \right)^2 \right)^{(1+\alpha)/2}.
\]

Since
\[
\left( \text{sign}^+ \left( \sum_{v_j \in \mathcal{N}_i} a_{ij}(x_j-x_i) \right) \sum_{v_j \in \mathcal{N}_i} a_{ij}(x_j-x_i) \right)^2 \geq 0
\]
following the result of Lemma 3 in [14], we have
\[
\sum_{i=1}^{n} \left( \left( \text{sign}^+ \left( \sum_{v_j \in \mathcal{N}_i} a_{ij}(x_j-x_i) \right) \sum_{v_j \in \mathcal{N}_i} a_{ij}(x_j-x_i) \right)^2 \right)^{(1+\alpha)/2} \geq \left( \sum_{i=1}^{n} \sum_{v_j \in \mathcal{N}_i} a_{ij}(x_j-x_i)^2 \right)^{1+\alpha}/2.
\]

Hence, we have
\[
\frac{dV_1(t)}{dt} \leq -\sum_{i=1}^{n} \left( \left( \text{sign}^+ \left( \sum_{v_j \in \mathcal{N}_i} a_{ij}(x_j-x_i) \right) \sum_{v_j \in \mathcal{N}_i} a_{ij}(x_j-x_i) \right)^2 \right)^{(1+\alpha)/2}.
\]

Referring to Lemma 2 (note that \(1/2 < (\alpha + 1)/2 < 1\))
\[
\sum_{i=1}^{n} \left( \left( \text{sign}^+ \left( \sum_{v_j \in \mathcal{N}_i} a_{ij}(x_j-x_i) \right) \sum_{v_j \in \mathcal{N}_i} a_{ij}(x_j-x_i) \right)^2 \right)^{1+\alpha}/2 \geq \frac{1}{n} \sum_{i=1}^{n} \left( \sum_{v_j \in \mathcal{N}_i} a_{ij}(x_j-x_i)^2 \right)^{1+\alpha}/2.
\]

If \(V_1 \neq 0\)
\[
\frac{\sum_{i=1}^{n} \left( \sum_{v_j \in \mathcal{N}_i} a_{ij}(x_j-x_i) \right)^2}{V_1} = \frac{X^T L(A)^T L(A) X}{1/2 X^T L(A) X}.
\]

According to the properties of the graph Laplacian \(L(A)\)
\[
\frac{X^T L(A)^T L(A) X}{1/2 X^T L(A) X} \geq 2\lambda_2(L(A)).
\]

Finally
\[
\frac{dV_1}{dt} \leq \left( \frac{\sum_{i=1}^{n} \left( \sum_{v_j \in \mathcal{N}_i} a_{ij}(x_j-x_i) \right)^2}{n V_1(t)} \right)^{(1+\alpha)/2} V_1(t) \leq \left( \frac{2\lambda_2(L(A))}{n} \right)^{(1+\alpha)/2} V_1(t)^{(1+\alpha)/2}.
\]
According to Lemma 1, \( V_1(t) \) will reach 0 in finite time
\[
t^* \leq \frac{(2V_1(0))^{(1-\alpha)/2}}{(1-\alpha)(\frac{\lambda_2(L(A))}{n})^{(1+\alpha)/2}} \quad \square
\]

Remark 2. Note that Lyapunov function is assumed to be semi-definite instead of positive definite for the purpose of consensus problem (refer to [18–21] for details).

Remark 3. Compared to the protocol in [13], the protocol in Theorem 1 can be thought of as removing the negative values or positive values. In other words, the finite-time consensus is still guaranteed even if some information is lost. This fact provides the fault tolerance of the protocol with respect to actuator failures, as will be shown later.

Corollary 2. If communication topology \( G(A) \) is connected, then either one of the following protocols:
\[
u_i = \text{sat}^+ \left( \text{sign} \left( \sum_{j \in N_i} a_{ij}(x_j-x_i) \right) \sum_{j \in N_i} a_{ij}(x_j-x_i)^{\alpha} \right)
\]
\[
u_i = \text{sat}^- \left( \text{sign} \left( \sum_{j \in N_i} a_{ij}(x_j-x_i) \right) \sum_{j \in N_i} a_{ij}(x_j-x_i)^{\alpha} \right)
\]
(0 < \( \alpha \) < 1) solves a finite-time agreement problem.

Proof. The proof follows Lemma 2 and Theorem 1. \( \square \)

Remark 4. Compared to the protocol in [13], the protocol in Theorem 1 can be thought of as adding saturation terms on the inputs. In other words, the finite-time consensus is still guaranteed even if some actuators suffers saturation. This fact provides the fault tolerance of the protocol with respect to loss of effectiveness failure, as will be shown later.

3. Problem formulation of cooperative interception

In this section, the problem of multiple flight vehicles intercepting single moving target simultaneously is investigated. To decouple the task into two sub-tasks, the guidance law design is divided into two stages. The first stage is to design the command acceleration to guarantee the convergence of relative distance, while the second stage is to adjust the guidance law to guarantee that all vehicles intercept the target simultaneously. In particular, the second stage is formulated as a finite-time consensus problem.

The engagement geometry for a planar interception between one flight vehicle and the constant-velocity target is shown in Fig. 1 where \( M \) denotes flight vehicle and \( T \) denotes target. The line-of-sight (LOS) denotes the line between \( M \) and \( T \), hence, we construct LOS frame whose origin is fixed to \( M \) and two axes point at LOS direction and normal LOS direction. Since most measurement information from the sensor is convenient to be described in the LOS frame, we formulate the relative kinematics in the LOS frame as
\[
u_r = r\dot{q}^2 - \ddot{r}
\]
\[
u_q = -\dot{r}q - 2\dot{r}\dot{q}
\]
where $r$ denotes the relative distance between the flight vehicle and the target, $q$ denotes the angle between LOS and the inertial reference line $x$, and $u_r$, $u_q$ are the acceleration components of the flight vehicle in the LOS frame. We introduce variables $V_r = \dot{r}$ and $V_q = r\dot{q}$ to denote the components of relative velocity in LOS frame, respectively.

By introducing state variable $[r, V_r, V_q]^T$, we can transform Eqs. (17) and (18) into a nonlinear differential equation

$$
\begin{bmatrix}
\dot{r} \\
\dot{V}_r \\
\dot{V}_q
\end{bmatrix} =
\begin{bmatrix}
V_r \\
\frac{V_r^2}{r} - u_r \\
-\frac{V_r V_q}{r} - u_q
\end{bmatrix}.
$$

We introduce a variable, termed time-to-go, to represent the time required for interception. It is a dynamic variable, as it is time-decay. Since in a practical interception scenario with traditional guidance law, the variation of the closing-velocity $V_r$ is relatively small, the time-to-go can be approximated as

$$
t_{go} = -\frac{r}{V_r}
$$

To investigate the variation of time-to-go with respect to the relative motion information, the derivative of time-to-go can be obtained as

$$
i_{go} = -1 + \frac{V_q^2}{V_r^2} - \frac{r}{V_r^2} u_r
$$

The multi-to-one interception with $n$ flight vehicles and one constant-velocity target is illustrated in Fig. 2, where the initial condition of each individual flight-target pair is allowed to be different from others. The objective of the cooperative interception is to design the guidance law (i.e., to design $u_r$ and $u_q$) for each vehicle, ensuring that all the flight vehicles intercept the moving target simultaneously under the distributed communication topology. In order for time consistence, their $t_{go,i}$ must be kept the same through information exchange under the communication topology.

We use subscript $i$ represent the index of vehicle. To take $t_{go,i}$ as an extra state variable, the kinematics between flight vehicle $i$ and the target in the LOS frame can be described as

$$
\begin{bmatrix}
\dot{r}_{i} \\
\dot{V}_{r,i} \\
\dot{V}_{q,i} \\
\dot{t}_{go,i}
\end{bmatrix} =
\begin{bmatrix}
V_{r,i} \\
\frac{V_{r,i}^2}{r_{i}} - u_{r,i} \\
-\frac{V_{r,i} V_{q,i}}{r_{i}} - u_{q,i} \\
-1 + \frac{V_{q,i}^2}{V_{r,i}^2} - \frac{r_{i}}{V_{r,i}^2} u_{r,i}
\end{bmatrix}.
$$
The cooperative interception includes two tasks. An interception requires that \(|V_{q,i}| \to 0\) which implies \(r \to 0\) in finite time when \(V_{r,i} < 0\). In addition, the cooperation requires \(|t_{go,i} - t_{go,j}| \to 0\) in finite time, since time-to-go of individual flight vehicle must reach the agreement of the group before the engagement.

To introduce a new control input

\[
\pi_{r,i} = \frac{V_{q,i}^2}{V_{r,i}^2} - \frac{r_i}{V_{r,i}^2} u_{r,i},
\]

the design of \(u_{r,i}\) can be transformed into the design problem of \(\pi_{r,i}\), since the state variables \(V_{q,i}\), \(V_{r,i}\) and \(r_i\) can be measured or calculated in real time. In a practical guidance process, \(|V_{q,i}|\) is much smaller than \(|V_{r,i}|\), which implies \(V_{q,i}^2 / V_{r,i}^2\) is very close to zero. Therefore, we can assume that \(\pi_{r,i}\) has the opposite sign with the original control input \(u_{r,i}\).

Substituting Eq. (23) into Eq. (22), it yields

\[
\begin{bmatrix}
\dot{r}_i \\
\dot{V}_{r,i} \\
\dot{V}_{q,i} \\
\hat{t}_{go,i}
\end{bmatrix}
= \begin{bmatrix}
V_{r,i} \\
V_{q,i}^2 / r_i \pi_{r,i} \\
- r_i V_{q,i} - u_{q,i} \\
-1 + \pi_{r,i}
\end{bmatrix}.
\]

Traditional guidance law designs impose the control input \(u_{q,i}\) on the normal direction of the LOS which guarantee the velocity \(V_{q,i}\) approaching zero. Since the variation of the closing velocity is relatively small, once initial closing velocity is smaller than zero, control input \(u_{r,i}\) is set zero during the approach process. However, the dynamic of time-to-go \(t_{go,i}\) depends on the other state variables and control input \(u_{r,i}\) in view of Eq. (21). Using the actuator on the LOS direction to change the time-to-go is feasible.

In view of Eq. (24), the process of guidance law design can be separated into two stages. The first stage is to design command acceleration \(u_{q,i}\) which guarantees velocity \(V_{q,i}\) goes to zero. Here, a traditional proportional navigation guidance in [16] is adopted for each individual flight vehicle to guarantee the convergence of \(V_{q,i}\). Command acceleration \(u_{q,i}\) of flight vehicle \(i\) can be
shown as \[16\]

\[ u_{q,i} = -\frac{N_i V_{r,i} V_{q,i}}{r_i} \]  

where \(N_i\) represents the navigation constant of proportional navigation guidance.

Based on the first stage, the design can be simplified as the control for the following equation:

\[ \dot{t}_{go,i} = -1 + \pi_{r,i}. \]

Thus, the second stage is to design command acceleration \(\pi_{r,i}\) which guarantees that \(t_{go,i}\) goes to the agreement value of the team. In other words, we need to design a finite-time consensus protocol to guarantee \(t_{go,i}\) are in an agreement. The reason we have to consider finite-time consensus, rather than other asymptotic consensus, is that the consensus must be reached before the engagement time. In addition, different from the finite-time consensus problem in Section 2, consensus variable \(t_{go}\) in our cooperative interception problem is dynamic, rather than static. This makes the consensus more complicated.

### 4. Cooperative interception with actuator failures

In this section, we will propose a novel cooperative interception with tolerance to actuator failures of the participating vehicles. The following theorem proposes a distributed controller which guarantees the consensus of time-to-go. During a guidance process, the flight vehicle can calculate time-to-go according to current relative information and predict the engagement instant. Hence, we can define the predictive engagement instant for the flight vehicle which represents the calculated engagement instant at time \(t\). Before we proposed the consensus protocols for cooperative interception, we made several assumptions for the purpose of simplifying the problem.

**Assumption 1.** \(V_r\) has relatively small variation compared to other variables and is assumed as a negative constant.

**Assumption 2.** \(V_{q}^2/V_r^2\) is close to zero compared to other variables is assumed as zero.

**Assumption 3.** The angle between the LOS direction and the axial direction of the body of the vehicle is zero.

**Theorem 3.** If communication topology \(G(A)\) of multiple flight vehicles is connected, then the protocol

\[ \pi_{r,i} = \text{sign} \left( \sum_{v_j \in N_i} a_{ij}(t_{go,j} - t_{go,i}) \right) \sum_{v_j \in N_i} a_{ij}(t_{go,j} - t_{go,i})^\alpha \]  

realizes a finite-time agreement of time-to-go.

**Proof.** By introducing a new variable \(t_{e,i} = t + t_{go,i}\) to represent the predictive engagement instant at time \(t\) of flight vehicle \(i\), we have

\[ \dot{t}_{e,i} = \pi_{r,i}. \]  

Therefore, the finite-time agreement of time-to-go is equivalent to that of predictive engagement instant.
By Theorem 1 in [13], consensus protocol

\[ \pi_{r,i} = \text{sign} \left( \sum_{v_j \in N_i} a_{ij}(t_{e,j} - t_{e,i}) \right) \sum_{v_j \in N_i} a_{ij}(t_{e,j} - t_{e,i})^\alpha \]

solves the finite-time agreement problem and consensus time

\[ t^* \leq \frac{(2V_1(0))^{(1-\alpha)/2}}{(1-\alpha)(\lambda_2(L(A)))^{(1+\alpha)/2}} \]

where the same Lyapunov candidate \( V_1(t) \) with Eq. (14) is chosen and \( 0 < \alpha < 1 \).

Since \( t_{go,j} - t_{go,i} = t_{e,j} - t_{e,i} \), the theorem can be shown. \( \square \)

For convenience, we define Eq. (26) as consensus protocol 1 in the rest of this paper.

**Remark 5.** It can be seen from Theorem 3 that for each vehicle, the protocol relies on its time-to-go, as well as those from the neighbors. According to last section, time-to-go can be computed based on the dynamics of the vehicle.

One important issue associated with this protocol is the implementation, i.e., to implement \( u_{r,i} \) on each vehicle. It is related to the actuators in the vehicle. In the practical interception, the angle between the LOS direction and the axial direction of vehicle body is so small due to the limit of the seeker that it can be approximated as zero. Hence, control input \( u_{r,i} \) can be provided by installing motors on the head and tail of the body. However, due to the physical constraints, tail motor can only provide positive \( u_{r,i} \). In contrast, head motor can only provide a negative \( u_{r,i} \). Correspondingly, only positive \( \pi_{r,i} \) can be produced if only the head motor works, and only negative \( \pi_{r,i} \) can be produced if only the tail motor works. Therefore, to guarantee the implementation of the protocol in Theorem 3, both head and tail motors are required since \( \pi_{r,i} \) in Theorem 3 could be negative or positive.

However, due to the complicated environment in the air, the failure of physical devices and the fuel limit, these head (tail) motors are subject to some failures. A question is what kind of failures can the protocol tolerate.

The first case we investigate is that some head (or tail) motors fail completely. When the head (or tail) motor of flight vehicle \( i \) fails to work, the protocol input will be described as follows:

\[ \pi_{r,i} = \text{sign}^+ \left( \sum_{v_j \in N_i} a_{ij}(t_{go,j} - t_{go,i}) \right) \sum_{v_j \in N_i} a_{ij}(t_{go,j} - t_{go,i})^\alpha \]

\[ \times \left( \text{or } \pi_{r,i} = \text{sign}^- \left( \sum_{v_j \in N_i} a_{ij}(t_{go,j} - t_{go,i}) \right) \sum_{v_j \in N_i} a_{ij}(t_{go,j} - t_{go,i})^\alpha \right) \]

In other words, the fault model can be described as that a head (or tail) motor fails to work and cannot provide control input. Then the above protocol cannot be implemented. However, if we impose the protocol inputs for these failed motors to be zeros, then we still can guarantee consensus. Specifically, the following theorem states that the protocol (26) can realizes a cooperative interception simultaneously, when some of head (or tail) motors fail to work.

**Theorem 4.** If communication topology \( G(A) \) of multiple flight vehicle is connected, then the protocol as Eq. (26) realizes a finite-time agreement of time-to-go when \( n_f \) head (or tail) motors
fail to work where \( n_f = 1, 2, \ldots, n \). Here, we assume that for these failed motors, the inputs are assumed to be zero.

**Proof.** If there is no failure, then the protocol in Theorem 3 is obtained. When there are some head (or tail) failures, according to the assumption, we let the inputs be zeros. Thus, we obtain a new protocol as stated in Theorem 1, since \( \text{sign}(-) \) in Theorem 3 becomes \( \text{sign}^+(-) \) or \( \text{sign}^-(−) \). Specifically, if only head failures happen, then we have \( \text{sign}^-(−) \). If only tail failures happen, we have \( \text{sign}^+(-) \). Therefore, following Theorem 1, the time-to-go consensus is still guaranteed. The detailed proof is as follows.

We choose the Lyapunov candidate

\[
V_1 = \frac{1}{4} \sum_{i,j=1}^{n} a_{ij} (t_{go,j} - t_{go,i})^2.
\]

Hence we have

\[
\frac{dV_1}{dt_{go,i}} = -\sum_{i \in J_f} \left( \text{sign}^+ \left( \sum_{v_j \in N_i} a_{ij} (t_{go,j} - t_{go,i}) \right) \right) \left( \sum_{v_j \in N_i} a_{ij} (t_{go,j} - t_{go,i}) \right)^2 (1+\alpha)/2
\]

\[
- \sum_{i \notin J_f} \left( \sum_{v_j \in N_i} a_{ij} (t_{go,j} - t_{go,i}) \right)^2 \left( \frac{1}{\alpha} \right)^{1+\alpha}
\]

where \( J_f \) consists of failure motor's index.

Since \( n_f \leq n \)

\[
\frac{dV_1}{dt_{go,i}} \leq -\sum_{i=1}^{n} \left( \text{sign}^+ \left( \sum_{v_j \in N_i} a_{ij} (t_{go,j} - t_{go,i}) \right) \right) \left( \sum_{v_j \in N_i} a_{ij} (t_{go,j} - t_{go,i}) \right)^2 (1+\alpha)/2.
\]

The following process will follow Theorem 1. The consensus will be reached in finite time

\[
t^* \leq \frac{(2V_1(0))^{(1-\alpha)/2}}{(1-\alpha)(\lambda_2(LA))^{(1+\alpha)/2}}.
\]

Note that when \( n_f = n \), protocol (26) still realizes a finite-time agreement of time-to-go, which implies the consensus of time-to-go can still be guaranteed if all head (or tail) motors on the flight vehicles are lost.

**Remark 6.** Although the protocol can guarantee the consensus, there is a price for the loss of some motors. According to the consensus time without considering loss of motor

\[
t^* \leq \frac{(2V_1(0))^{(1-\alpha)/2}}{(1-\alpha)(\lambda_2(LA))^{(1+\alpha)/2}}.
\]

the consensus time will increase under failures.

Another common failure in vehicles is the loss of effectiveness. In other words, some motors may happen to have limited operator range (or called saturation failure) due to some physical problems. In other words, a head (or tail) motor may still work, but has an amplitude limit due to
fuel limit or device failure. For this case, we impose the following modification for the protocol: if a saturation happens on one motor, then the protocol input for this motor is changed as $u = \text{sat}(u)$. The following theorem states protocol (26) can realize a cooperative interception, when there exist saturations on some of head (or tail) motors.

**Theorem 5.** If communication topology $G(A)$ of multiple flight vehicles is connected, then the protocol (26) realizes a finite-time agreement of time-to-go when there exist amplitude limits on $n_f$ head (or tail) motors where $n_f = 1, 2, \ldots, n$.

**Proof.** The proof can be deduced from Corollary 2 and Theorem 3 easily. □

**Remark 7.** Note that the amplitude limits between different head (tail) motors do not have to be same.

**Remark 8.** Protocol (26) may not guarantee the finite-time consensus of time-to-go when both the head motor failures and tail motor failures happen. It is clear by considering a simple case that a faster flight vehicle with only tail motor and a slower flight vehicle with only head motor. Since tail motor can only accelerate the speed and the head motor can only decelerate the speed, it is obvious that they could not reach the target simultaneously.

5. Cooperative interception within network failures

In a practical interception mission, the communication channels are not always reliable as a static topology due to the device limits, obstacles, and electromagnetic interference. Therefore, the consensus protocol should have certain tolerance of the network failures. Here, the fault model for network failure is that some vehicles are loss of communication with each other so that they cannot obtain the flight information from each other such as interception time. Mathematically, if vehicle $i$ losses the connection with vehicle $j$, then $a_{ij} = a_{ji} = 0$, $t_{\text{go},i}$ is unknown to vehicle $j$ and vice versa. We have the following theorem.

**Theorem 6.** Suppose the dynamic communication topology $G(A(t))$ is undirected and connected all the time. If $a_{ij}(t) = a_{ji}(t)$ and $0 < a_{ij}(t) < 1$ for all $i, j, t$, then consensus protocol

$$\pi_{r,i} = \sum_{j \in N_i} a_{ij}(t) \text{sign}(t_{\text{go},j} - t_{\text{go},i}) |t_{\text{go},j} - t_{\text{go},i}|^{a_{ij}}$$

(29)

realizes the cooperative interception and consensus engagement time $t^*_{e}$ is equal to $1/n \sum_{i=1}^{n} t_{\text{go},i}(0)$.

**Proof.** The predictive engagement instant at time $t$ is introduced as $t_{e,i} = t + t_{\text{go},i}$. According to the statement of Theorem 1 in [14], the protocol

$$\pi_{r,i} = \sum_{j \in N_i} a_{ij}(t) \text{sign}(t_{e,j} - t_{e,i}) |t_{e,j} - t_{e,i}|^{a_{ij}}$$

(30)

can solve the finite-time average-consensus problem of engagement instant.

Assuming the initial instant of cooperative guidance is zero, the finite-time average-consensus problem of engagement time means $t_{e,j}(t) \to 1/n \sum_{k=1}^{n} t_{e,k}(0)$ for all $j = 1, 2, \ldots, n$ as $t \to t^*$, where $t^*$ is consensus time of the predictive engagement instant.
Defining $t^e_e$ as the consensus engagement time, we have

$$t^e_e = \frac{1}{n} \sum_{i=1}^{n} t_{e,i}(0) = \frac{1}{n} \sum_{i=1}^{n} t_{go,i}(0).$$

where $t_{e,i}(0) = t_{go,i}(0) + 0$.

Replacing $t_{e,j} - t_{e,i}$ with $t_{go,j} - t_{go,i}$, Theorem 6 is shown.

We define Eq. (29) as consensus protocol 2 in the following part.

**Theorem 6.** Suppose that $\mathcal{G}(A(t))$ is undirected and connected with time intervals. The sum of time intervals, in which $\mathcal{G}(A(t))$ is unconnected, is less than the maximum limit, i.e.,

$$T_{\text{max}} = \frac{1}{n} \sum_{i=1}^{n} t_{go,i}(0) - \frac{2^{1-\alpha_0} V_2(0)^{(1-\alpha_0)/2}}{1-\alpha_0)K^{(1+\alpha_0)/2}}. \quad (31)$$

If $\alpha_{ij}(t) = \alpha_{ji}(t)$ and $0 < \alpha_{ij}(t) < 1$ for all $i,j,t$, then protocol as Eq. (29) solves the finite-time consensus problem of time-to-go.

**Proof.** Referring to Theorem 1 in [14], protocol (29) solves the finite-time average-consensus problem of engagement instant.

According to Theorem 6, the engagement time can be calculated by the initial time-to-go estimations as $t^e_e = 1/n \sum_{i=1}^{n} t_{go,i}(0)$. To guarantee all the flight vehicles intercept the target simultaneously, the consensus instant should be smaller than engagement time $t^e_e$. We assume that the unconnected topology is produced by network failures and the communication topology is always connected except the time intervals with network failures.

According to the proof of Theorem 1 in [14], the sum of time intervals in which the topology is connected is larger than

$$T_{\text{connected}} = \frac{2^{1-\alpha_0} V_2(0)^{(1-\alpha_0)/2}}{(1-\alpha_0)K^{(1+\alpha_0)/2}} \quad (32)$$

where

$$V_2(t) = \frac{1}{2} \sum_{i=1}^{n} \left( x_i(t) - \frac{1}{n} \sum_{j=1}^{n} x_j(t) \right)^2, \quad \alpha_0 = \max_{i,j} \alpha_{ij},$$

$$K = \min \{ \bar{K}(t) \bar{L}(B(t)) : \mathcal{G}(A(t)) \text{ is connected.} \}$$

$$B = \left[ a_{ij}^{2/(1+\alpha_0)} \right]_{n \times n} \quad \text{and}$$
\[ \hat{K} = \frac{1}{\sum_{i,j=1}^{n} a_{ij}^2/(1+a_{ij})} \min_{a_{ij} \neq 0} a_{ij}^{2/(1+a_{ij})} \left( \max_{k} t_{go,k}(0) - \min_{k} t_{go,k}(0) \right)^{2/(1+a_{ij})/(1+a_{ij}) - 1}. \]

Since a necessary condition of cooperative interception is the consensus should be realized before the engagement, the sum of time intervals should be smaller than \( t^{*} - T_{\text{connected}}. \]

Theorem 7 gives the tolerance degree of consensus protocol (29) under the network failures. By estimating the maximum unconnected-topology interval, we can judge whether consensus protocol (29) can be applied into the cooperative interception with certain initial conditions.

6. Simulations

We consider a three-to-one scenario, i.e., three flight vehicles of a team to intercept one moving target. The vehicles exchange the real-time flight information through the consensus protocol under a given communication topology. We consider four different communication topologies of the team, as shown in Fig. 3, in which the first three ones are connected and the last one is unconnected. If flight vehicle \( i \) can communicate with flight vehicle \( j \), then \( a_{ij} = 1 \), otherwise, \( a_{ij} = 0 \), \((i,j = 1,2,3)\). Thus, the four topologies imply different \( a_{ij} \), and thus different communications in protocols. These flight vehicles have their dynamics, i.e., Eq. (24). The initial conditions between flight vehicles and the single moving target are assumed to be different, as listed in Table 1.

To consider the cooperative interception, for each flight vehicle, we divide the guidance law into two stages, according to Section 3. In the first stage, we design \( u_{q,i} = N_i V_{r,i} V_{q,i}/r_i \) with navigation constant \( N_i = 3 \) to make \( V_{q,i} \) convergent to zero [16]. For the second stage, it becomes a finite-time consensus problem of time-to-go.

For our proposed consensus protocols (protocols 1 and 2), nine cases are considered to test their tolerance under different failures, as described in Table 2. In each case, three variables with respect to time \( t \) are investigated since they are the critical variables on the performance of cooperative interception. They are \( r_i \), \( V_{q,i} \) and \( t_{go,i} \). Here, \( r_i \) represents the time instant that flight vehicle \( i \) intercepts the target, \( V_{q,i} \) represents whether the interception of flight vehicle \( i \) is successful, and \( t_{go,i} \) represents the required time for the flight vehicle \( i \) to intercept the target.

In case \( \circ \), we do not implement any consensus protocol of time-to-go on all flight vehicles and we assume there is no failure. The variations of three investigated variables are shown in Fig. 4. It can be seen that due to no consensus protocol, the time-to-go and interception time among vehicles are independent. In particular, their \( t_{go,i} \)s almost have the same decay rate. Obviously, the cooperative interception fails to exist.

The engagement scenarios with finite-time consensus protocol 1 (\( \alpha = 0.5 \)) are investigated from case \( \odot \) to case \( \circ \), considering the actuator failures as the impact factor of cooperative interception. The communication topology is assumed as static connected topology 2, as shown in Fig. 3. In case \( \odot \), we assume there is not actuator failure. Fig. 5 shows all the variables with respect to time \( t \). It can be seen from Fig. 5(a) that all three flight vehicles intercept the target simultaneously. The same fact is also verified by Fig. 5(c), where the final time-to-gos are the same. In particular, it can be seen from Fig. 5(c) that these vehicles have different \( t_{go,i} \)s at time instant 0. However, Compared to case \( \odot \), due to the implementation of consensus protocol, they will have different decay rate of time-to-go. After around 2 s, they will reach the same value.
and then have the same decay rate 1. In other words, the time-to-go reaches consensus.

We consider case ③, that is, the tail-motor of flight vehicle 3 fails. The variations of corresponding variables with respect to time $t$ are shown in Fig. 6. It can be seen that the

Table 1
Initial condition between flight vehicle $i$ and single moving target ($i=1,2,3$).

<table>
<thead>
<tr>
<th>$M_i \rightarrow T$</th>
<th>Initial condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_1 \rightarrow T$</td>
<td>$[r_i(0), V_{d_i}(0), V_{r_i}(0), t_{go_i}(0)]^T$</td>
</tr>
<tr>
<td>$M_3 \rightarrow T$</td>
<td>$[2404.2, -91.9, 35.3, 26.2]^T$</td>
</tr>
</tbody>
</table>

Table 2
Descriptions of different test cases.

<table>
<thead>
<tr>
<th>Case</th>
<th>Consensus protocol</th>
<th>Situation</th>
</tr>
</thead>
<tbody>
<tr>
<td>①</td>
<td>No consensus protocol</td>
<td>Without any failures</td>
</tr>
<tr>
<td>②</td>
<td>Consensus protocol 1</td>
<td>Without any actuator failures</td>
</tr>
<tr>
<td>③</td>
<td>Consensus protocol 1</td>
<td>With tail-motor failure on flight vehicle 3</td>
</tr>
<tr>
<td>④</td>
<td>Consensus protocol 1</td>
<td>With head-motor failures on all flight vehicles</td>
</tr>
<tr>
<td>⑤</td>
<td>Consensus protocol 1</td>
<td>With head-motor saturation failure on flight vehicle 1</td>
</tr>
<tr>
<td>⑥</td>
<td>Consensus protocol 1</td>
<td>With tail-motor saturation failures on all flight vehicles</td>
</tr>
<tr>
<td>⑦</td>
<td>Consensus protocol 2</td>
<td>With switching connected topology all the time</td>
</tr>
<tr>
<td>⑧</td>
<td>Consensus protocol 2</td>
<td>With long unconnected-topology intervals</td>
</tr>
<tr>
<td>⑨</td>
<td>Consensus protocol 2</td>
<td>With short unconnected-topology intervals</td>
</tr>
</tbody>
</table>

(around 22 s in Fig. 5(c)), and then have the same decay rate 1. In other words, the time-to-go reaches consensus.

We consider case ③, that is, the tail-motor of flight vehicle 3 fails. The variations of corresponding variables with respect to time $t$ are shown in Fig. 6. It can be seen that the
consensus protocol 1 still guarantee all the flight vehicles intercept the target simultaneously. However, compared to no failure (case ②), the consensus time for time-to-go is longer. We also consider the extreme situation (case ③), where all three head-motors broke down. Fig. 7 shows the variations of three variables with respect to time t. It can be seen that a simultaneous interception can also be realized. However, compared to no failure case (case ②), the consensus time for time-to-go increases to 6 s. It verifies our conclusion that failure may not affect the finite-time consensus, but may deteriorate the consensus time. For this case, since all the vehicles can only reduce the time-to-go, the convergence time of cooperative interception for time-to-go reduces too. Thus, the final convergence time (around 21 s in this case) is smaller than no failure case. Note that this does not mean that the head failure case will bring better performance, because smaller convergence time is not necessarily the best indicator of better performance. On the contrary, in many cases, convergence time range that represents the time duration to reach consensus is a better one. It is obvious that with the header failure, the final convergence time range becomes small, which may imply the worst performance.

Saturation failures are considered in cases ⑤ and ⑥. Here, the motor of flight vehicle does not break down completely, but has an amplitude limit due to fuel limit or device failure. We use semi-saturation of control input to describe this kind of failure. In case ⑤, we assume that flight vehicle 1 provides positive 0.5 m/s² acceleration maximally. Fig. 8 shows the variations of the three variables. It can be seen that all flight vehicles intercept the target simultaneously. However, there is an increase of consensus time of time-to-go, compared to no failure case. In Case ⑥, we assume that all tail-motors can only provide negative 0.5 m/s² acceleration maximally. Fig. 9 shows the validity of consensus protocol 1. Compared to case ⑤, the consensus value increases from 23 s to 25 s.

![Fig. 4. Case ①: no consensus protocol and without any failure. (a) Variations of r with respect to time t. (b) Variations of Vq with respect to time t. (c) Variations of tgo with respect to time t.](image)

![Fig. 5. Case ②: consensus protocol 1 without any actuator failure. (a) Variations of r with respect to time t. (b) Variations of Vq with respect to time t. (c) Variations of tgo with respect to time t.](image)
The engagement scenarios with finite-time consensus protocol 2 (\(\alpha_j = 0.5\)) are investigated in case \(\mathbb{F}\) to case \(\mathbb{H}\), where the network failure is thought of as the impact factor of cooperative interception. We assume that the communication topology switches between the connected ones and unconnected one as shown in Fig. 3. In case \(\mathbb{F}\), we assume that the connection maintains all the time. The variations of three variables with respect to time \(t\) are shown in Fig. 10. It can be seen that all the three flight vehicles can intercept the target simultaneously under switching connected topology.

In cases \(\mathbb{G}\) and \(\mathbb{H}\), we will test the tolerance of the protocol 2 to the communication failure in terms of unconnected topology. By Theorem 7, the maximum tolerance of unconnected topology is estimated as 10.8 s. So in case \(\mathbb{G}\), we assume the topology is unconnected for 20 s, longer than the maximum tolerance. Fig. 11 shows the variations of the three variables with respect to time \(t\).
It can be seen from Fig. 11(a) that one flight vehicle intercept the target at a different time from the other two. Meanwhile, Fig. 11(c) demonstrates that the consensus of time-to-go for three vehicles fails to be reached.

In case ⑨, we shorten the unconnected interval to 5 s. The variations of the three variables are shown in Fig. 12. It can be seen that a simultaneous interception is obtained by consensus protocol 2. Comparison of cases ⑧ and ⑨ demonstrates the existence of the communication failure tolerance, as stated in Theorem 7.

The control inputs $u_r$ for the cases ②–⑦ are also plotted in Fig. 13, which show certain chattering. This unexpected chattering could deteriorate the system performance to some extent.
However, the effect would not be too much because it has a relatively small magnitude and always disappears with the completion of consensus. To reduce or even eliminate the chattering phenomenon will further improve this work.

7. Conclusions and future works

The problem of fault tolerant control in cooperative interception using multiple flight vehicles was investigated in this paper. Considering time-to-go as the time-decay agreement variable, the cooperative interception was formulated as the finite-time consensus problem of multi-agent system. Actuator failures and network failures in the cooperative interception were analyzed. One finite-time consensus protocol was adopted in the situations where there existed actuator failures. The first finite-time consensus protocol was proved to have certain tolerance against actuator failures of flight vehicles. The second finite-time consensus protocol was also adopted.

Fig. 12. Case ⑨: consensus protocol 2 with short unconnected-topology intervals. (a) Variations of $r$ with respect to time $t$. (b) Variations of $V_q$ with respect to time $t$. (c) Variations of $t_{go}$ with respect to time $t$.

Fig. 13. Variations of $u_r$ with respect to time $t$. (a) $u_r$ in case ②. (b) $u_r$ in case ③. (c) $u_r$ in case ④. (d) $u_r$ in case ⑤. (e) $u_r$ in case ⑥. (f) $u_r$ in case ⑦.
when there existed network failures. According to the calculation of the settling time, the maximum tolerance against network failures was estimated via the proposed method.

There are some future studies related to this work. One interesting study is to the finite-time consensus protocol with both head motor failures and tail motor failures. In addition, the finite-time consensus protocol with undirected topology that represents a wide class of network failures is also an interesting topic. Finally, our protocols include sign functions, which may cause chattering phenomenon. How this phenomenon affects the system performance as well as how to reduce it is an interesting topic.

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References

