Guarding a Territory Against an Intelligent Intruder: Strategy Design and Experimental Verification

Han Fu, Hugh H.T. Liu

Abstract—**This paper designs and tests a dominant region based strategy for a group of defenders to intercept an intruder before it enters a target area. The intruder is intelligent in the sense that it makes decisions based on the defenders' strategies instead of following a pre-defined path, making the problem a differential game. The proposed strategy is optimal when the intruder moves slower than the defenders. When the intruder moves faster, the dominant region based strategy is still applicable, but with proper modifications. Crazyflie 2.1 is used as the experimental platform to test the proposed strategy. For fixed defender locations, a barrier line can be drawn such that the intruder is guaranteed to be captured outside the target area if it starts beyond.**

Index Terms—**differential game, counter-UAV, dominant region, Crazyflie**

I. Introduction

As research efforts and developments in unmanned aerial vehicles (UAVs) increase, concerns about the potential security threats also grow. Drones have been reported shown up in prohibited or sensitive areas, such as government buildings, military bases, and airports. For example, in 2015, a drone crashed near the White House. In 2018, the UK Airprox Board reported that a drone intentionally crashed into an aircraft. The number of near-misses between drones and aircraft has risen from 6 events to 93 during 2014 to 2017.

Such concerns lead to the development of the counter-UAV technology, which is dedicated to detect and intercept drones. It has been used for airspace protection at airports, security during large events such as party conventions and sports games, VIP protection, and counter smuggling operations at prisons, etc. [1]

The counter-UAV scenario can be modeled as a guarding territory differential game, where an intruding drone tries to enter a target area, while a group of defenders seek to intercept it. The defenders need to move in proper directions so as to get the intruder in the range of their interception devices.

The guarding territory game is a special case of pursuit-evasion games where the intruder (corresponding to the evader in a pursuit-evasion game) also wishes to enter a target area, in addition to escaping from the defender (pursuer). The earliest and simplest guarding territory game was formulated by Isaacs, where a defender protects a circular target area against one intruder. [2]

The standard way of solving a pursuit-evasion game is to construct and solve the Hamilton-Jacobi-Bellman-Isaacs (HJI) equation, [3] which is hard to solve in general due to the curse of dimensionality. In order to incorporate the target, a special modification is needed. [4] The modified HJI equation contains four min/max operators, making the problem more challenging.

Efforts have been made in dimension reduction and incorporating geometric methods. Chen et al. decomposed a 4-dimensional problem into 2-dimensional slices, and presented a conservative strategy for defenders. [5] Makkapati et al. studied sub-problems for different pursuit strategies, where the state spaces were carefully chosen such that the HJI equations were solvable. [6] Kawkecki et al. [7] and Rzymowski [8] solved the maximal length of a line segment that one and multiple defenders can protect respectively, by constructing a set of geometric auxiliary functions.

While auxiliary functions in [7] [8] were customized for the specific problems, a more general geometric concept is called the dominant region (DR), which is a set of points that one player can reach before the other players. [2] The dominant region has been extensively used in differential game problems for strategy design, [9]–[11] optimality proof, [12] and player assignment in multi-player problems. [9] The capability of the dominant region has also been extended into games with obstacles and uncertainties. [13], [14]

Many of the works, however, assume that the capture range is zero, [12], [13] or the pursuers travel faster than the evaders, [15], [16] or the target area is to some extend symmetric. [10], [16] This paper attempts to relax such limitations. First of all, a non-zero capture range is considered, which allows the defenders to take advantage of the large effective range of some counter-UAV techniques. Secondly, the speed limit on the intruder is eliminated, and both cases are covered that the intruder travels slower and faster than the defenders. Finally, the proposed strategy is applicable for any convex target area.

The essence of the proposed strategy is to find the closest point to the target area from the intruder's dominant region. When the defenders travel faster, the dominant region based strategy is equivalent to the classic solution. When the defenders are slower, however, the classic solution is open-loop, [17] and should be customized for different targets. This paper approximates the key property of the classic optimal trajectory with a simple function, making the proposed strategy closed-loop and applicable in more general situations, with little loss of optimality.

The effectiveness of the proposed strategy is tested through experiment, which is rarely done in researches concerning differential games.

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II. PROBLEM FORMULATION

The scenario is illustrated in Figure 1. Suppose function $g(\boldsymbol{x})$ is convex and bounded from below, and the convex target area can be described as $\mathcal{A} = \{\boldsymbol{x}|g(\boldsymbol{x}) \leq 0\}$. Because the value of $g(\boldsymbol{x})$ reflects how far point $\boldsymbol{x}$ is out from or how deep it is in the target, we'll refer to it as the target level.

The group of defenders form a blockade around the target, while the intruder seeks to pass through the defenders and enter the target. Assume the intruder is captured once it enters any defender's interception range. i.e., $\|ID_i\| < r, \exists i \in \mathbb{D}$, where $\mathbb{D}$ is the index set of the defenders.

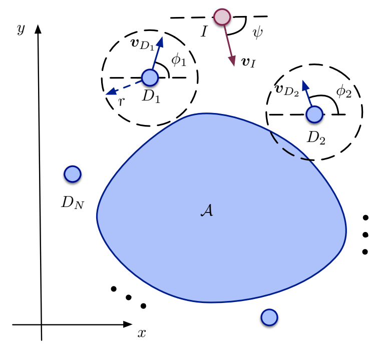


Fig. 1: Problem formulation of the guarding territory game

The kinematics of the system is described by

$$\begin{aligned} \dot{x}_I &= v_I \cos\psi, & \dot{y}_I &= v_I \sin\psi \\ \dot{x}_{D_i} &= v_{D_i} \cos\phi_i, & \dot{y}_{D_i} &= v_{D_i} \sin\phi_i,\ i \in \mathbb{D} \end{aligned} \tag{1}$$

where $v_k, k \in \{I\} \cup \mathbb{D}$ are velocities of the players. ψ, $\phi_i, i \in \mathbb{D}$ are heading angles and control inputs. Assume that v_I and $v_{D_i} = v_D, i \in \mathbb{D}$ are constant, and define speed ratio $a = v_D/v_I$. In the following discussion, we'll use $\boldsymbol{p}_I = (x_I, y_I)$ and $\boldsymbol{p}_{D_i} = (x_{D_i}, y_{D_i})$ to represent player locations, and refer to $\boldsymbol{x} = (\boldsymbol{p}_I, \boldsymbol{p}_1, \cdots, \boldsymbol{p}_N) \in \mathbb{R}^{2(N+1)}$ as the state.

Define the regulated capture time $t_c = \min\{\bar{t}_c, t_e\}$, where $t_e = \min\{t|\boldsymbol{p}_I(t) = \arg\min_{\boldsymbol{p}} g(\boldsymbol{p})\}$ is the first time that the intruder reaches the center of the target, where the target level is minimized. This regulation keeps t_c finite when the intruder cannot be captured.

The defenders' goal is to find strategies $\{\phi_i^*(\boldsymbol{x}), i \in \mathbb{D}\}$ that maximize the target level of $\boldsymbol{p}_I(t_c)$, while the intruder hopes to find strategy $\psi^*(\boldsymbol{x})$ that minimizes the same value. Assume that the min and max operations are commutable, the game can be described by the minmax problem:

$$\min_{\psi} \max_{\phi_i, i \in \mathbb{D}} g(\boldsymbol{p}_I(t_c)) \tag{2}$$

Before presenting the solution, we state an important fact about the optimal trajectories in Theorem 1, which has been proved for $r = 0$, [12] yet is generally true for kinematics that contain no state variable on the right-hand-side and when $\|ID_i\| > r, \forall i \in \mathbb{D}$. The optimal trajectory of player $j, j \in \{I\} \cup \mathbb{D}$ refers to function $\boldsymbol{p}_j(t)$ that is solved from (1) when the optimal strategies ψ^*, ϕ_i^* are adopted.

Theorem 1: The optimal trajectories of the guarding territory game (2) are straight lines when $\|ID_i\| > r, \forall i \in \mathbb{D}$.

III. STRATEGIES FOR $a \geq 1$

When $a \geq 1$, a geometric solution of (2) exists based on the concept of dominant regions.

A. Dominant Regions

The dominant region of a player is the set of points it can reach before any other players. For one intruder I and one defender D ($\|DI\| > r$), the dominant region of the intruder is $\mathcal{D}_I^D = \{P|\|PD\| - r \geq a\|PI\|\}$, and the dominant region of the defender is its complementary, i.e., $\mathcal{D}_D^I = \mathcal{D}_I^{D^C}$.

When two players are considered, the dominant region of the slower player is bounded. When the intruder travels no faster than the defender, its dominant region $\mathcal{D}_I^D$ is convex, as shown in Figure 2.

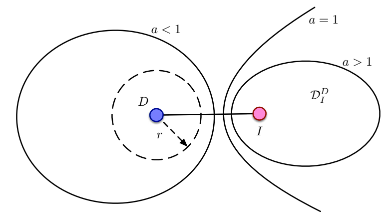


Fig. 2: Dominant regions of one intruder against one defender

When there are multiple defenders, the overall dominant region of the intruder is the intersection of all dominant regions with respect to each defender, $\mathcal{D}_I = \cap_{i\in\mathbb{D}} \mathcal{D}_I^{D_i}$, and is convex when $a \geq 1$, because each $\mathcal{D}_I^{D_i}$ is convex.

B. The Optimal Strategy

With the dominant region defined, the guarding territory game with $a \geq 1$ can be re-formulated as an equivalent optimization problem:

$$\begin{aligned} &\min_{\boldsymbol{p}} g(\boldsymbol{p}) \\ &s.t.\ \boldsymbol{p} \in \mathcal{D}_I, \quad \mathcal{D}_I = \cap_{i\in\mathbb{D}} \mathcal{D}_I^{D_i} \end{aligned} \tag{3}$$

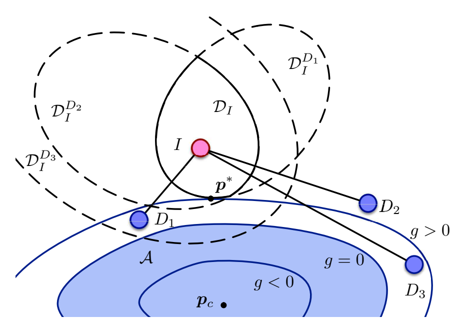


Fig. 3: The equivalent optimization problem when $a \geq 1$

Due to the convexity of $\mathcal{D}_I$ and g, problem (3) is convex and has a unique solution, denote by $\boldsymbol{p}^*$. It indicates the 'closest' point to or the 'deepest' place in the target area that the intruder can reach. Because the optimal trajectories are straight lines, the intruder's optimal strategy is to head to $\boldsymbol{p}^*$ directly.

Suppose the target is large enough such that the center of the target, $\boldsymbol{p}_c$, is not contained in $\mathcal{D}_I$, then $\boldsymbol{p}^*$ is at the boundary of $\mathcal{D}_I$, as shown by Figure 3. The best that the defenders can do is to head towards $\boldsymbol{p}^*$ along straight lines, so as to capture the intruder there and ensure that the intruder gets no closer to the target than $\boldsymbol{p}^*$.

When there are multiple defenders, $\boldsymbol{p}^*$ can be out of some defenders' dominant regions. For example in Figure 3, $\boldsymbol{p}^* \notin \mathcal{D}^I_{D_3}$. In this case, D_3 is redundant. D_1 will capture the intruder before D_3 arrives.

In the blockade, all the other defenders except the two intermediate neighbors of the intruder are redundant, e.g., D_3 in Figure 3. As a result, the multi-defender single-intruder guarding territory game can be reduced to a two-defender problem. The intruder only needs to pass through the two closest neighbors on its left and right.

Suppose the ith defender is not redundant, by definition of the dominant region, $(\|D_i\boldsymbol{p}^*\| - r)/\|I\boldsymbol{p}^*\| = v_D/v_I$. During the play, lengths $\|D_i\boldsymbol{p}^*\| - r$ and $\|I\boldsymbol{p}^*\|$ reduce at speeds of v_D and v_I, hence point $\boldsymbol{p}^*$ doesn't change and the trajectories are straight lines. By Theorem 1 and the property of $\boldsymbol{p}^*$, the dominant based strategy is the minmax solution to (2).

C. Winning Conditions and the Barrier

The winner of the game is determined by the location of $\boldsymbol{p}^*$. If $\boldsymbol{p}^* \in \mathcal{A}$, the intruder wins because it can get in the target before being captured. If $\boldsymbol{p}^* \notin \mathcal{A}$ the intruder loses because it will be captured before entering the target. If $\boldsymbol{p}^* \in \partial\mathcal{A}$, the game has a neutral outcome and the intruder will be captured on the boundary of the target.

Consider the two-defender game, if locations of the two defenders are fixed, initial locations of the intruder that lead to the neutral outcome form a curve called the barrier. The intruder loses if it starts beyond, and wins if starts below. More discussions will be presented in Section IV-E and Section V.

IV. STRATEGIES FOR $a < 1$

When the intruder moves faster, it cannot be captured by a single defender because it has the ability to at least maintain a fixed distance from one defender. As shown in Figure 4, the intruder can spare part of his velocity to match that of the defender, and use the rest to rotate around the defender. This strategy is referred to as the distance maintaining strategy.

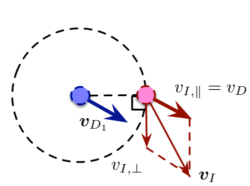


Fig. 4: The intruder's distance maintaining strategy

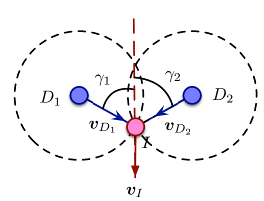


Fig. 5: Players' relative locations upon capture

Therefore, the intruder must be captured by at least two coordinated defenders simultaneously, as shown in Figure 5. Denote by $\gamma_i = |\angle(\boldsymbol{v}_I, \boldsymbol{v}_{D_i})|, \gamma_i \in [0, \pi]$ the angle between the intruder and the ith defender's velocity vectors. When $\|ID_i\| = r, i = 1, 2$, the changing rate of the distance between each defender and the intruder is $\|I\dot{D}_i\| = v_I \cos\gamma_i - v_{D_i}$. It decreases with γ_i, and is positive when $\gamma_i < \arccos a \triangleq \gamma_0$, zero when $\gamma_i = \gamma_0$ and negative when $\gamma_i > \gamma_0$. For this reason, larger γ_i is preferable for defender i upon capture.

According to Theorem 1, when $a > 1$, as long as none of the capture regions is reached, the optimal trajectories are still straight lines. In the event that one of the capture regions is reached, the intruder can at least maintain this distance. Although the intruder has the capability of moving away from that defender, it should not do so, because that way it will get closer to other defenders. Having similar distances from multiple defenders is not beneficial for the intruder, because capture needs to be achieved by more than one defenders.

To prevent defenders from cooperating, and at the same time avoid capture, the intruder should conduct the distance maintaining strategy. Trajectories of the intruder and the closest defender thus start to curve. As a result, the game is divided into two stages, Phase I that all the optimal trajectories are straight lines, and Phase II that the optimal trajectories of the intruder and the closest defender are curved. [17]

The standard way of solving the game is in a backward manner, i.e., Phase II is solved first, based on which the slopes of the straight-line trajectories in Phase I can be obtained. But the solution of Phase II varies with the shape of the target area, and is generally hard to solve. Further, such solution is a set of open-loop optimal trajectories, from which deducing a closed-loop strategy is tricky.

To obtain a feedback strategy, we apply the concept of dominant regions. For simplicity, we will be focused on two-defender games. But the proposed strategy can be extended to multiple defenders easily, since the intruder eventually has to pass through a certain pair of defenders.

A. The Target Approaching Strategy

When the intruder is faster, capture is not guaranteed. We first present a necessary condition for the intruder to win.

As shown in Figure 6, denote by T_i the point of tangency of $\mathcal{D}^I_{D_i}$ from I. Safe directions for the intruder to go is within sector T_1IT_2. Hence, the intruder's winning condition requires vector $\overrightarrow{IT_2}$ to be counter clockwise to $\overrightarrow{IT_1}$. i.e., $\angle T_1IT_2 > 0$.

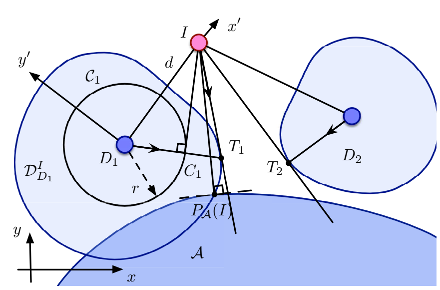


Fig. 6: The target approaching strategy of the intruder

Under this condition, a good choice for the intruder is to find the direction in sector T_1IT_2 that leads to the shortest path to the target.

A simple way to compute direction $\overrightarrow{IT_i}$ can be deduced from the following theorem.

Theorem 2: Given defender D_i and intruder I ($\|D_iI\| > r$), $\mathcal{D}^I_{D_i}$ is the defender's dominant region, T_i is the point of tangency from I to $\mathcal{D}^I_{D_i}$, $\mathcal{C}_i$ is the boundary of the defender's capture region. Then the following facts hold:

1) $\cos\angle T_iD_iI = r/\|D_iI\|$,
2) connect D_iT_i, which intersects $\mathcal{C}_i$ at C_i. $IC_i \perp D_iT_i$,
3) $\angle IT_iD_i = \gamma_0 = \arccos a$.

Proof: Define reference frame $x'D_iy'$ as shown in Figure 6. The defender D_i is at the original point, and the intruder is at $(d, 0)$. The boundary of the defender's and the intruder's dominant regions, $\partial\mathcal{D}^I_{D_i}$ ($= \partial\mathcal{D}^{D_i}_I$), can be described by

$$\sqrt{x'^2 + y'^2} - r = a\sqrt{(x' - d)^2 + y'^2} \tag{4}$$

Differentiate Equation (4) with respect to x', we have

$$\frac{x' + y'dy'/dx'}{\sqrt{x'^2 + y'^2}} = a\frac{(x' - d) + y'dy'/dx'}{\sqrt{(x' - d)^2 + y'^2}} \tag{5}$$

where dy'/dx' is the slope of the tangent line at point (x', y') on $\partial\mathcal{D}^I_{D_i}$, which should be equal to the slope of line IT_i. So the function of line IT_i is

$$y' = \frac{dy'}{dx'}(x' - d) \tag{6}$$

Combine Equation (5)(6) and use (4), we have

$$\cos\angle T_iD_iI = \frac{x'}{\sqrt{x'^2 + y'^2}} = \frac{r}{d}$$

This proves 1), and 2) is a direct consequence of 1).

To prove 3), we first rewrite Equation (4) as

$$\|D_iT_i\| - r = a\sqrt{\|D_iT_i\|^2 + d^2 - 2\|D_iT_i\|d\cos\angle T_iD_iI} \tag{7}$$

from which $\|D_iT_i\|$ can be solved as

$$\|D_iT_i\| = r + a\sqrt{\frac{d^2 - r^2}{1 - a^2}} \tag{8}$$

According to the definition of the dominant region, we have

$$\|T_iI\| = \frac{1}{a}\left(\|D_iT_i\| - r\right) = \sqrt{\frac{d^2 - r^2}{1 - a^2}} \tag{9}$$

Therefore $\cos\angle IT_iD_i$ can be computed from the cosine law,

$$\cos\angle IT_iD_i = \frac{\|D_iT_i\|^2 + \|T_iI\|^2 - \|D_iI\|^2}{2\|D_iT_i\|\|T_iI\|} \tag{10}$$

With Equation (8)(9) and $\|D_iI\| = d$, one can easily check that $\cos\angle IT_iD_i = a$, which proves 3). ■

With Theorem 2, T_i can be obtained by simply rotating ID_i by $\angle D_iIT_i = \arcsin r/\|D_iI\| + \pi/2 - \gamma_0$. In the two-defender game, such rotation is counter clockwise for D_1 and clockwise for D_2, assuming D_1 is on the left and D_2 is on the right of the intruder, as shown in Figure 6.

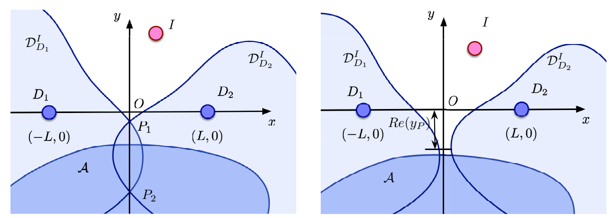
The target approaching strategy is only for the intruder. The defenders' strategy under $\angle T_1IT_2 > 0$ is the dominant region based strategy described in the following section.

B. The Dominant Region Based Strategy

When $\angle T_1IT_2 \le 0$, the winner of the game is uncertain. We introduce the dominant region based strategy with a slight modification. First define a new reference frame as shown in Figure 7. The original point is at the middle of segment D_1D_2, and the x-axis is along vector $\overrightarrow{D_1D_2}$. The two defenders are at $(\pm L, 0)$, while the intruder is at (x, y). Dominant regions of the two defenders against the intruder, $\mathcal{D}^I_{D_1}$ and $\mathcal{D}^I_{D_2}$, can be solved by definition.

Consider the situation where $\mathcal{D}^I_{D_1}$ and $\mathcal{D}^I_{D_2}$ intersect. In order to reach out for the intruder simultaneously, and because all the defenders share the same speed, a good choice for the two defenders is to head to a point on the y-axis, denote by $(0, y_P)$. Therefore,

$$a\sqrt{x^2 + (y - y_P)^2} = \sqrt{L^2 + y_P^2} - r \tag{11}$$



(a) $\mathcal{D}^I_{D_1} \cap \mathcal{D}^I_{D_2} \neq \varnothing$ (b) $\mathcal{D}^I_{D_1} \cap \mathcal{D}^I_{D_2} = \varnothing$

Fig. 7: The dominant region based strategy for $a < 1$

Equation (11) has two roots. When $\mathcal{D}^I_{D_1} \cap \mathcal{D}^I_{D_2} \neq \varnothing$, both roots are real, denote by y^1_P and y^2_P. Each root represents an intersection point, denote by P_1 and P_2. Without loss of generality, assume $\|IP_1\| \le \|IP_2\|$, as shown in Figure 7a. Since the two defenders should capture the intruder simultaneously, they must head to a waypoint on segment P_1P_2. On this segment, P_1 is the best choice because it yields the largest γ_i, and has the minimum target level. For the intruder, P_1 is the closest point to the target within its dominant region, therefore it should move to P_1 as well.

When $\mathcal{D}^I_{D_1} \cap \mathcal{D}^I_{D_2} = \varnothing$, Equation (11) has a pair of complex roots, whose real parts indicate the location of the narrowest part of the intruder's dominant region, as shown in Figure 7b. The two defenders should head to point $(0, \mathrm{Re}(y_P))$ in hope of minimizing the distance to the intruder when it passes. The intruder should also move towards the same point in hope of breaking through safely.

C. The Proximity Strategy for Defenders

When initial locations are very asymmetric, e.g., x is large, one of the capture regions will be reached first, and the game switches to Phase II. In this case, the intruder will adopt the distance maintaining strategy, and the closer defender should apply some proximity strategy accordingly. Since Phase II is hard to solve, we design a simple strategy that imitates the behavior of the optimal trajectories in [17], where two pursuers seek to capture a faster evader, and the evader merely wants to escape in between, without aiming at any target are. The optimal trajectories of this game have a similar two-phase structure, as shown in Figure 8.

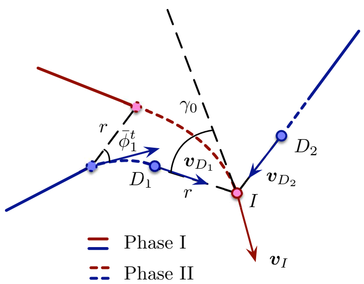


Fig. 8: The two-phase structure of the optimal trajectories of the two-pursuer single-evader game in [17]

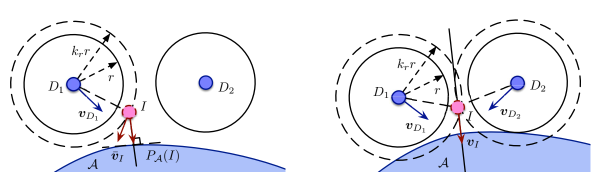
At the beginning of Phase II, the closer pursuer doesn't head towards the evader directly, but rather gradually turns into it as the game progresses. Because the distance maintaining strategy requires the evader to spare part of its velocity to match that of the closer pursuer, this pursuer takes advantage of it and forces the intruder towards the further pursuer in the hope of cooperation.

As the game goes on, the further pursuer gets closer, capture retains higher priority, the closer pursuer gradually allocates more speed along the line-of-sight of the evader. When capture happens, the pursuers' strategies become pure-pursuit.

In the guarding territory game, the defenders play similar roles to the pursuers. Assume D_1 to be the closer defender, and $\bar{\phi}_1^t$ the angle between its moving direction and vector $\overrightarrow{D_1 I}$ at time step t, we can write $\bar{\phi}_1^{t+1} = k_\phi \bar{\phi}_1^t$, where the discount rate $k_\phi \in (0, 1)$ is a complex function of the state. We set k_ϕ to be constant as a simple approximation of the increasing emphasis on capture. Simulations show that $k_\phi \approx 1$ are good choices.

D. The Proximity Strategy for the Intruder

The distance maintaining strategy of the intruder is risky. If the intruder kept a precise distance of r from the closer defender, it could be captured even with minor mistakes. Hence, we allow for a buffer zone and let the intruder switch to the distance maintaining strategy when it is $k_r r$ away from the closer defender, as shown in Figure 9a. The closer defender adopts the proximity strategy since then accordingly.



(a) $\|ID_1\| \le k_r r, \|ID_2\| > k_r r$ (b) $\|ID_1\| \le k_r r, \|ID_2\| \le k_r r$

Fig. 9: The intruder's proximity strategy

Except avoiding capture, the intruder also endeavors to enter the target area. Without loss of generality, assume D_1 to be the closer defender, as shown in Figure 9a. Denote by $\bar{\boldsymbol{v}}_I$ the velocity generated by the distance maintaining strategy, if it is clockwise to $\overrightarrow{IP_{\mathcal{A}}(I)}$, a better choice of the intruder is to move along the latter. Here $P_{\mathcal{A}}(I)$ is the projection of point I on set $\mathcal{A}$.

When $k_r > 1$, the intruder can be within the buffer capture zone of both defenders, as shown in Figure 9b. According to the trajectories in Figure 8, the optimal strategy of the intruder in this case is to move along the bisector of $\angle D_1 I D_2$, while the defenders should apply the pure-pursuit strategy.

E. Simulations

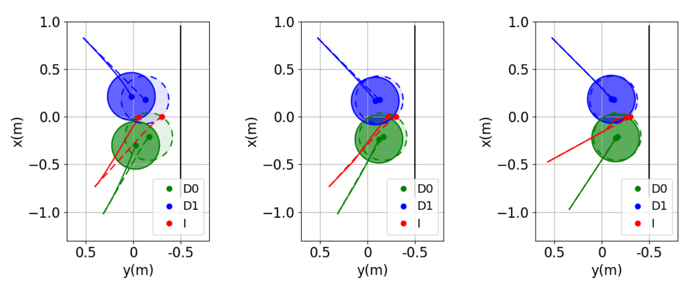
1) Comparison with the Classic Solution: When $a \ge 1$, the proposed strategy is a direct extension of Isaacs' guarding territory example [2] and is optimal. When $a < 1$, the proposed strategy is designed under the same concept, therefore we shall expect the results to be close to the classic optimal solution.

To solve the game with the classic approach, one should determine the terminal locations first, and trace backward to find the corresponding initial locations that the game starts from. What this solution presents is a set of optimal trajectories. Therefore the classic solution is only feasible when the initial locations coincide on these optimal trajectories.

To verify the proposed strategy, we computed the classic optimal trajectories to find feasible initial locations, and ran the game from there using the proposed strategy. The simulation was done for two defenders with speed $v_D = 0.25m/s$, a straight-line target $\mathcal{A} = \{(x, y) | y \le -0.5m\}$, a capture range of $r = 0.25m$, and the discount rate $k_\phi = 0.98$. The results are shown in Figure 10.

The trajectories of the proposed strategy (solid lines) are close to the classic solution (dashed lines), and the difference mainly comes from Phase II, because $\bar{\phi}_1^{t+1} = k_\phi \bar{\phi}_1^t$ is only a rough approximation.

To see this, the trajectories under buffer ratios $k_r = 1.2$ and $k_r = 1.0$ with the same initial locations are shown in Figure 10a and 10b. Because the intruder switches to the distance maintaining strategy earlier when $k_r = 1.2$, the deviation from the classic optimal trajectory is more significant.

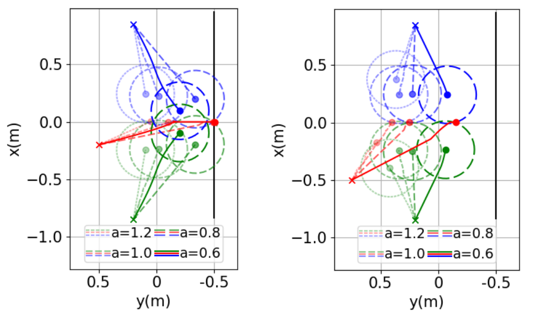


(a) $k_r = 1.2$, with Phase II (b) $k_r = 1.0$, with Phase II (c) $k_r = 1.2$, without Phase II

Fig. 10: Comparison with the classic optimal trajectories

In Figure 10a and 10b, the initial location of the intruder is much closer to one defender than the other, therefore it reaches the capture range of the closer defender first. As a result, the distance maintaining strategy is adopted, and Phase II exits. When the initial locations are less asymmetric, as the case in Figure 10c, capture ranges of the two defenders are reached simultaneously, the trajectories of the proposed strategy closely match that of the classic solution.

2) The Strategy under Different Speed Ratios: By varying the intruder's speed, we further compared the performance of the proposed strategy under different speed ratios, and the results are shown in Figure 11. As a decreases, the intruder travels faster, thus the defenders must retrograde more to block it, except for the situation that the intruder is going to win, where the defenders move more actively towards the intruder, shown by the solid trajectories in Figure 11a.



(a) $\boldsymbol{p}_I = (-0.2m, 0.5m)$ (b) $\boldsymbol{p}_I = (-0.5m, 0.75m)$

Fig. 11: Trajectories under the proposed strategy for different speed ratios

The barrier under different speed ratios are shown in Figure 11a. The initial locations of the two defenders are at $(\pm 0.85m, 0.2m)$, the same as Figure 11. The initial locations of the intruder for Figure 11a and Figure 11b are shown as the red dot and cross respectively. The barrier of smaller speed ratios are closer to the target.

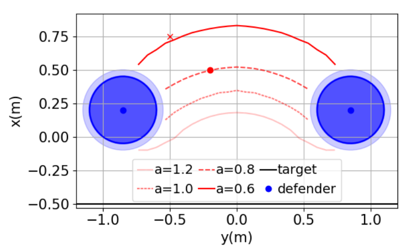


Fig. 12: The barrier under different speed ratios

V. EXPERIMENT RESULTS

A. Setup

The experiment was conducted on the Bitcraze Crazyflie 2.1 platform, the setup is shown in Figure 13. Each Crazyflie had onboard sensors and a microcontroller that stabilized the attitude, and was equipped with a communication system that received thrust and attitude commands.

The location and velocity information of the Crazyflies were measured by the OptiTrack system, a high-performance optical tracking system with sub-millimeter accuracy.[1] In the experiment, fourteen Flex 13 cameras were used and the data was processed by the Motive:Tracker software.

Data from the OptiTrack system was streamed to the central computer that ran the strategy, which took positions of all

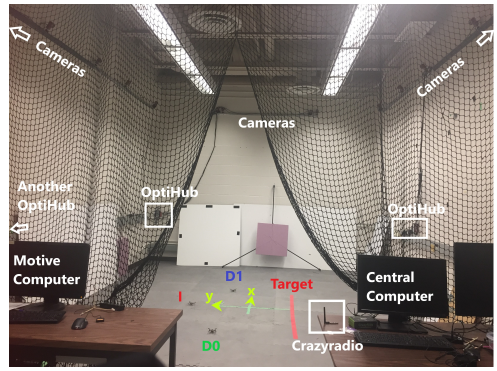


Fig. 13: The experiment setup

the Crazyflies and computed the heading angles. Because the magnitudes of velocities were constant, the heading angles were converted to velocity commands in x and y directions. The heights that the Crazyflies flew at were hold constant, different from each other to avoid collision. A proportional controller was used for altitude control.

The 2-dimensional velocity from the strategy and the vertical speed from the altitude controller were fed into a velocity controller to compute the required thrust and attitude, which were sent to the Crazyflies through Crazyradios, no direct communication among the Crazyflies was implemented.

A block diagram of the architecture is shown in Figure 14. The interface between the central computer and the Crazyflies were from the open source code provided by [18].

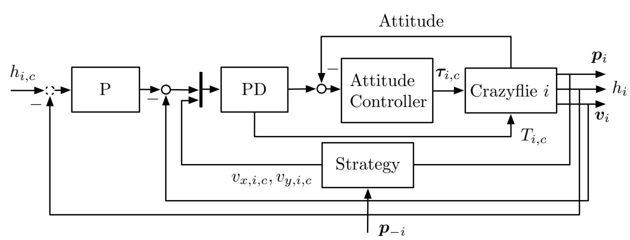


Fig. 14: The architecture of the Crazyflie 2.1 controller

The experiment was carried out for two defenders with $v_D = 0.25m/s$ and fixed initial locations $(\pm 0.85m, 0.2m)$. The target is the same as the simulation, and $k_r = 1.2$, $k_\phi = 0.98$.

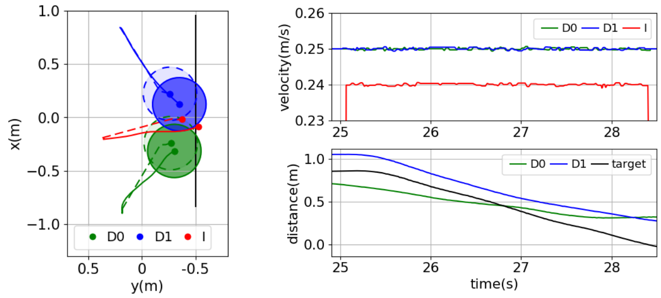
B. Slower Intruder

We first tested the case with $v_I = 0.24m/s$. Considering possible uncertainties of the experiment, we let both defenders play the game although one of them could be redundant.

With initial locations of the defenders fixed, we picked up a set of initial x-coordinates of the intruder $\{x_I^i\}$. For each x_I^i, we ran the experiment for different initial y-coordinates $\{y_I^j\}$, and observed if the intruder was captured or entered.

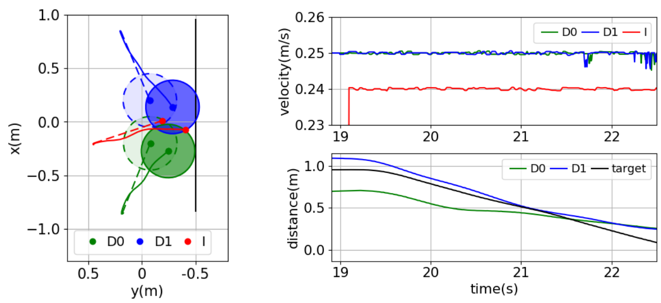
We initially tested each initial location twice. If the intruder was captured far from the target, and the two runs yielded same results, we labeled the corresponding initial location as 100% capture. If the defenders were far away when the intruder reached the target, and the two runs yielded same results, we

[1] https://v22.wiki.optitrack.com/index.php?title=Calibration#Calibration_Result_Report



(a) Trajectories (b) Velocities and distances

Fig. 15: Experiment results for $v_D = 0.25m/s$, $v_I = 0.24m/s$, $x_I = -0.2m$, $y_I = 0.4m$, the intruder wins



(a) Trajectories (b) Velocities and distances

Fig. 16: Experiment results for $v_D = 0.25m/s$, $v_I = 0.24m/s$, $x_I = -0.2m$, $y_I = 0.5m$, defenders win

labeled the corresponding initial location as 100% entering. If none of the conditions above were satisfied, we ran additional experiments with the identical initial locations and recorded the percentage of capture cases.

Figure 15 and 16 show representative cases for the 0% and 100% capture rate respectively. The dashed lines are simulation results with identical initial locations for comparison. As can be seen from the figures, experimental trajectories roughly followed the simulation results, but it costs the defenders longer distances to capture the intruder. The magnitudes of speeds of the players and the distances from the intruder to the defenders and the target are shown in Figure 15b and 16b.

The experiment showed that for each x_I^i, the capture rate decreased with the y-coordinate, and both 100% and 0% capture rate were observed. If some critical y-coordinate, y_I^*, had 50% capture rate, the initial location (x_I^i, y_I^*) was considered to be on the barrier. i.e., if the intruder started from initial locations vertically above (x_I^i, y_I^*), it would more likely be captured than enter the target. For each x_I^i, if the critical y-coordinate was not found from the experiment, it was estimated through linear interpolation.

Figure 17 shows the capture rate of different initial locations tested. Each point is represented by a pie chart, where the green (red) part shows the percentage of capture (entering). The barrier solved from simulation is shown as the red dashed line for comparison. Because defenders took longer distances to capture the intruder in the experiment, the barrier measured from the experiment is further away from the target area.

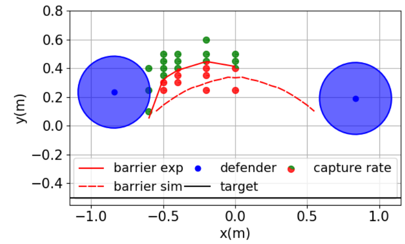
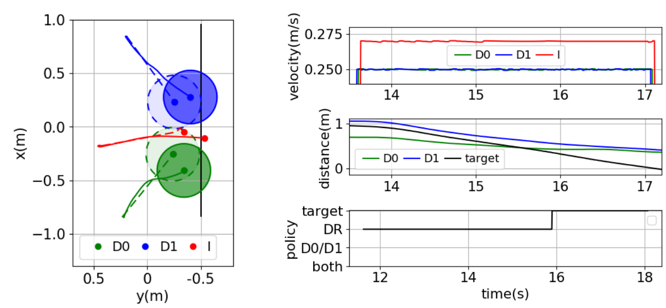


Fig. 17: The barrier for $v_D = 0.25m/s$, $v_I = 0.24m/s$

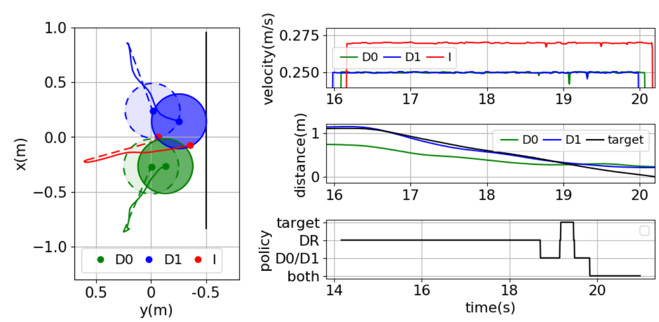
C. Faster Intruder

We increased the intruder's speed to $v_I = 0.27m/s$ and repeated the procedure above. Representative trajectories for entering and capture are shown in Figure 18 and 19 respectively. Similar to the slower intruder case, capturing the intruder was more difficult in the experiment.



(a) Trajectories (b) Velocities, distances and strategies

Fig. 18: Experiment results for $v_D = 0.25m/s$, $v_I = 0.27m/s$, $x_I = -0.2m$, $y_I = 0.5m$, the intruder wins



(a) Trajectories (b) Velocities, distances and strategies

Fig. 19: Experiment results for $v_D = 0.25m/s$, $v_I = 0.27m/s$, $x_I = -0.2m$, $y_I = 0.6$, defenders win

Since strategies for the faster intruder case are mixed, the intruder's strategies are also plotted. The four labels in the last plot of Figure 18b and 19b represent the target approaching strategy, the dominant region based strategy, the proximity strategies when one and both defenders are close.

In Figure 18, the intruder's winning condition was met and the target approaching strategy was adopted at around $15.9s$. In Figure 19, one of the buffer capture zones was reached around $18.6s$. After $0.4s$ of playing the distance maintaining strategy, the intruder's winning condition was met. But as being chased down by the defenders, the intruder had to switch back to the distance maintaining strategy. At around $19.4s$, the other defender arrives, and the intruder was captured eventually.

The barrier measured is shown in Figure 20, which is still above that of the simulation.

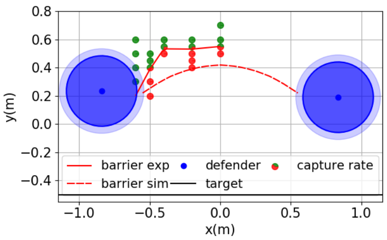


Fig. 20: The barrier for $v_D = 0.25m/s$, $v_I = 0.27m/s$

VI. CONCLUSIONS

This paper designs a dominant region based strategy for a group of defenders to intercept a single intruder before it enters a target area. This strategy allows the defenders to take advantage of their non-zero capture range, is able to handle an intruder with higher speed, and is applicable for any convex target. Simulation shows that the proposed strategy is close to the classic optimum, yet is more general and easier to use. The effectiveness of the proposed strategy was proved through experiment, although uncertainties and noises appeared to give advantages to the intruder.

The limitation of our work, however, is that the players are treated as point masses during the strategy design, with the assumption that the control inputs can be immediately followed. The impact of imperfect control of actual drones can be observed in the experiment, but not further studied. We also assume a game of perfect information, where each player's location is always available to all the other players.

Although rooted in an outdoor application, this paper only verified the proposed strategy through indoor experiment. In outdoor environments, localization results and communications will be subject to much larger uncertainties, and Crazyflies are not suitable for outdoor flights. Also, the buffer ratio k_r should be more carefully chosen for different environments, higher values are required for higher uncertainties.

Researches that address the aforementioned problems will be valuable works to improve our strategy, including incorporating quadrotor dynamics, considering imperfect information and uncertainties, verifying and improving the robustness of the proposed strategy, and conducting outdoor experiments.

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