Defending a Target Area with a Slower Defender

Han Fu, Hugh Liu
Presented by Pravin Wedage

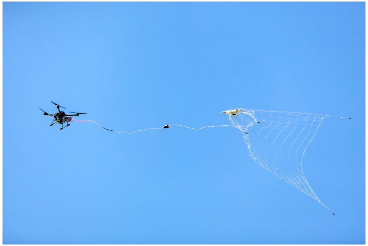
Flight Systems and Control Lab (FSC),
University of Toronto Institute for Aerospace Studies (UTIAS)

ACC 2023, May 31 - June 2, 2023 | San Diego, CA, USA

May 31, 2023

Outline

Background



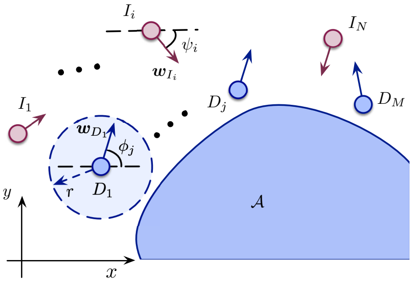
- Increasing concerns: drones at government buildings, military bases, airports, etc.
- Solution: to detect and intercept (counter-UAS technology)

2. Problem Description

2.1 Target Defense Games

Our interest

- Assume detection and interception are solved,
- How can the defending drones get the invaders in range?
- Can we identify how to place defenders?

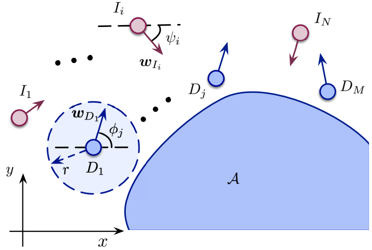


2. Problem Description

2.1 Target Defense Games

Assumptions

- Invaders are smart: *the problem becomes a game*.
- The distance the players travel are much larger than their physical sizes: *single integrator dynamics*.
- Perfect information on positions and velocities.
- Non-zero capture ranges ($r > 0$).



2. Problem Description

2.1 Target Defense Games

Goals of the players:

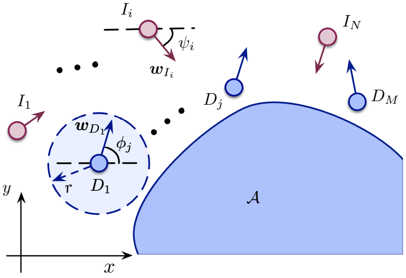
- Invaders: entering without capture.
- Defenders: prevent the entering.

System dynamics:

$$\dot{\boldsymbol{x}}_{D_j} = \boldsymbol{w}_{D_j}, \quad ||\boldsymbol{w}_{D_j}|| = U_D,$$
$$\dot{\boldsymbol{x}}_{I_i} = \boldsymbol{w}_{I_i}, \quad ||\boldsymbol{w}_{I_i}|| = U_I.$$

About "optimal":

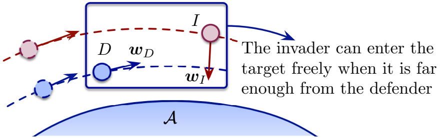
- A player would lose if changes were made.
- The solution handles the worst case.

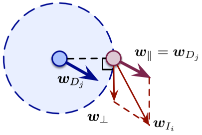


2. Problem Description

2.2 Slower Defender Makes A Difference

A faster invader is not guaranteed to be captured!



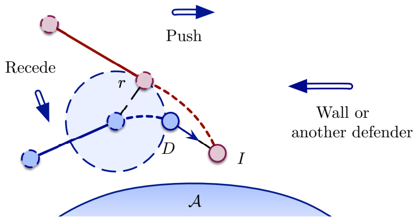


- The invader can enlarge the distance and enter the target area.
- The invader spare part of its velocity to match the defender, and uses the rest to rotate around (loop-around stage).

2. Problem Description

2.2 Slower Defender Makes A Difference

The optimal trajectory has a unique structure!



The game has two stages:

- Stage I: straight lines until capture range is reached.
- Stage II: the invader maintains the distance from the defender.

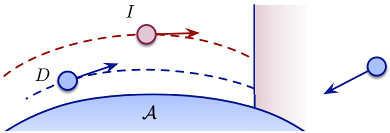
The defender balances two aspects:

- Push the invader along the perimeter of the target area.
- Recede toward the target area.

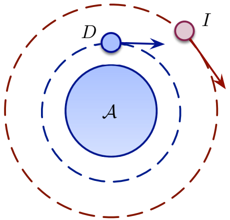
2. Problem Description

2.3 Two Cases Where a Slower Defender Can Win

Slower defender still has a chance to win!



- The game region is bounded, the defender can push the invader to the wall

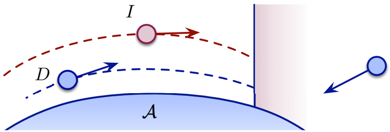


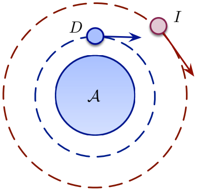
- The target area is small, the players end up rotating around the target area

2. Problem Description

2.3 Two Cases Where a Slower Defender Can Win

In both cases, there is a balance between two aspects:





For defender:

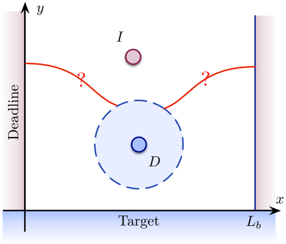
- Push the invader along the perimeter.
- Recede toward the target area.

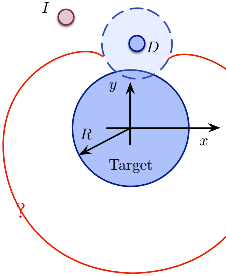
For invader:

- Enlarging the distance.
- Approaching the target area.

2. Problem Description

2.3 Two Cases Where a Slower Defender Can Win



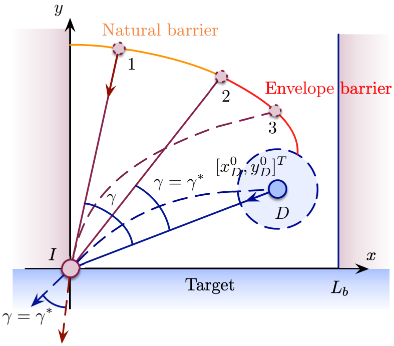


- **Game I:** the game region is bounded by two walls (deadlines)
- **Game II:** the target area is a small enough circle

Our goal: given defender location, solve for the barrier

3. Solutions

3.1 Solution of Game I



Solution depends on how the game ends:

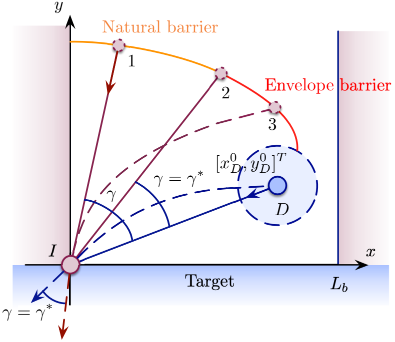
- The invader must be captured at the corner.
- The angle between the final velocities (γ) must be no less than $\gamma^* = \arccos(U_D/U_I)$.

Structure of the barrier:

- Natural barrier: when no loop-around stage exists.
- Envelope barrier: when the loop-around stage exists.

3. Solutions

3.1 Solution of Game I



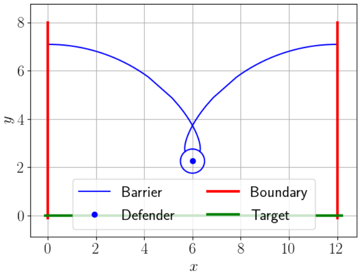
How is the barrier solved:

- Fix the defender location.
- Change the switching point between the two stages, the invader location gives the envelope barrier.
- Change γ, the invader location gives the natural barrier.

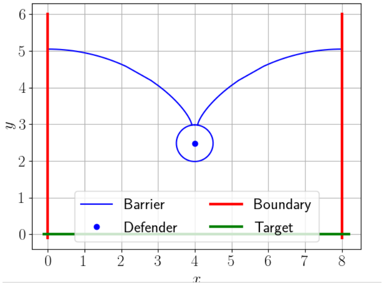
3. Solutions

3.1 Solution of Game I

- Defender position 1:

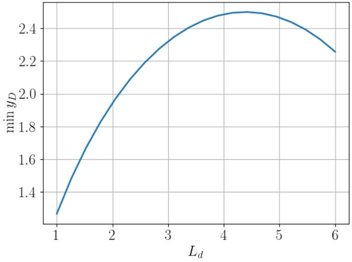


- Defender position 2:



3. Solutions

3.1 Solution of Game I



The following two variables are related:

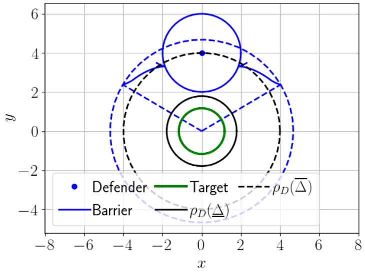
- The distance between the deadlines (L_d).
- The defender's minimum distance from the target area to not lose the game ($\min y_D$).

This solution gives a guideline to allocate the defenders!

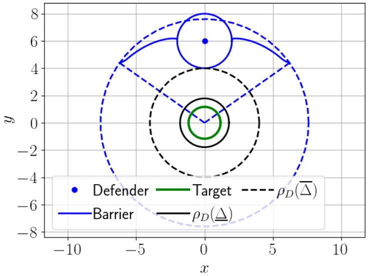
3. Solutions

3.2 Solution of Game II

- Defender position 1:

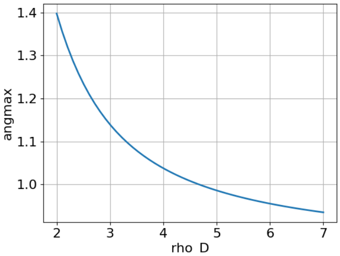


- Defender position 2:



3. Solutions

3.2 Solution of Game II



The following two variables are related:

- maximum portion of the circle that can be covered (angmax)
- defender's distance from the target area (ρ_D)

This solution gives a guideline to allocate the defenders!

Conclusion:

- This paper discusses the possibility of defending a target area with a slower defender and found two possible situations where it can successfully defend
- For both cases, the barrier is solved, which gives a guideline to allocate defenders

SAN DIEGO ACC 2023

Thank you!

Follow us on LinkedIn!

University of Toronto
Institute for Aerospace Studies
4925 Dufferin Street
Toronto, Ontario, Canada
Email:
pravin.wedage@flight.utias.utoronto.ca
Website:
www.flight.utias.utoronto.ca