

# A UDE-based Controller with Targeted Filtering for the Stabilization of a Fixed-Wing UAV in the Harrier Maneuver

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# Background



- Despite long range/endurance, many small UAV applications use multirotor vehicles over fixed-wing vehicles due to better low-speed control
- Solution: expand low speed operating envelope using **aerobatic maneuvers** to allow use of fixed-wings in applications that require both long range and low speed

## 2. Problem Description

### 2.1 Low Speed, High Angle-of-Attack Aerobatic Maneuvering

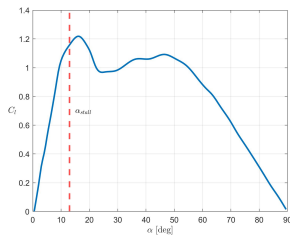
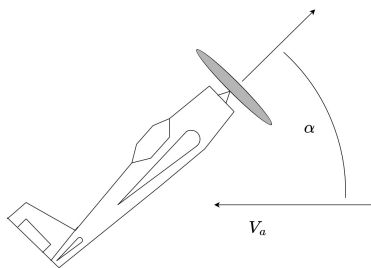


Fig 1: Lift coefficient vs angle-of-attack for a symmetric 2D airfoil.

- At lower airspeed, fixed-wing vehicles must increase angle-of-attack to maintain lift and altitude
- Below stall speed, small UAVs can maintain altitude using thrust. However, dynamics are more nonlinear, and control is more challenging

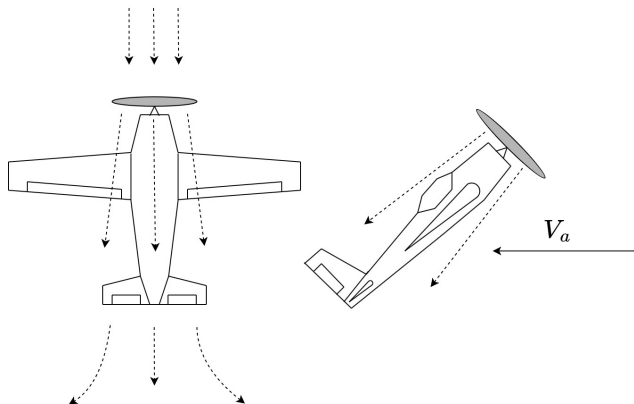


## 2. Problem Description

### 2.1 Low Speed, High Angle-of-Attack Aerobatic Maneuvering

#### Dynamics Challenges

- Nonlinear lift response
- Varying local velocities across the airframe
- Aerodynamic interaction between lifting surfaces
- Time-dependent effects
- Many of these features are difficult to accurately model



## 2. Problem Description

### 2.2 Time-Varying Effects During Maneuvering

- Time-varying dynamic effects and disturbances present
- High angle-of-attack results in varying lift, drag due post-stall phenomena on airfoils
- Additional dynamic modes observed at high angle-of-attack

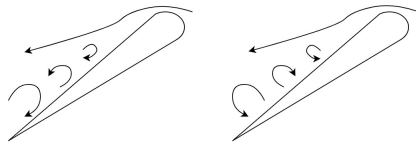


Fig 2: Vortices shed with alternating rotation directions over time, causing a fluctuating lift coefficient.

## 2. Problem Description

### 2.2 Time-Varying Effects During Maneuvering

- Typical wind disturbances have more pronounced effect during high angle-of-attack
- Velocity perturbations can cause sudden changes in attitude

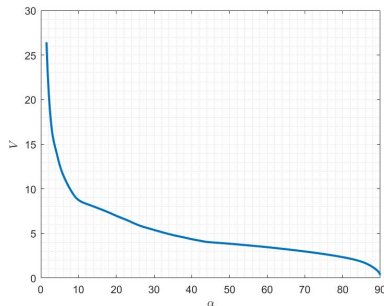


Fig 3: Airspeed velocity to maintain altitude for fixed-wing aerobatic UAV at a specific angle-of-attack.

## 2. Problem Description

### Summary of problem:

- Model uncertainty
- Nonlinearities
- Time-varying effects

Robust control solution to address all of these?

## 3. Control Solution

### 3.1 Uncertainty and Disturbance Estimator

#### Uncertainty and Disturbance Estimator (UDE) capabilities:

- Estimate and compensate for disturbances, given a nominally stabilizable system
- Two DOF freedom in designing nominal controller  $u_0$  and disturbance estimator  $\hat{d}$
- Lumped disturbance estimation, with frequency based estimation capabilities

$$\dot{x} = f(x, u) + d \quad (1)$$

$$u = u_0 - \hat{d} \quad (2)$$



## 3. Control Solution

### 3.1 Uncertainty and Disturbance Estimator

#### Application to this problem:

- Nominal aircraft system typically performs well with linearized models when controlling for fixed-operating points
- Propwash keeps aircraft inboard wing, fuselage, tail in linear aerodynamic response range
- Linear control solution developed with suitable derivation of aircraft longitudinal and lateral linearized model during aerobatic maneuvers

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{d} \quad (3)$$

$$\mathbf{B}\mathbf{u} = \mathbf{B}\mathbf{u}_0 - \hat{\mathbf{d}} \quad (4)$$

$$\mathbf{B}\mathbf{u} = -\mathbf{B}\mathbf{K}\mathbf{x} - \hat{\mathbf{d}} \quad (5)$$

$$\mathbf{A} \in \mathbb{R}^{n \times n}$$

$$\mathbf{B} \in \mathbb{R}^{n \times m}$$



## 3. Control Solution

### 3.1 Uncertainty and Disturbance Estimator

#### Design of disturbance estimator:

- Disturbance estimate obtained by filtering system dynamics
- Typical applications design filter in the frequency domain based on disturbance frequency

$$\mathbf{G}_f(s) = [g_{ij}] \in \mathbb{C}^{n \times n}$$

$$g_{ij} = 0, i \neq j$$

$$g_{ij} \neq 0, i = j$$

$$\hat{\mathbf{d}} = \mathbf{g}_f * \mathbf{d} \quad (6)$$

$$\hat{\mathbf{D}} = \mathbf{G}_f(s)\mathbf{D} \quad (7)$$

$$= \mathbf{G}_f(s) (s\mathbf{X} - \mathbf{A}\mathbf{X} - \mathbf{B}\mathbf{U}) \quad (8)$$

$$= ([\mathbf{I} - \mathbf{G}_f(s)]^{-1} \mathbf{G}_f(s)) \times (s\mathbf{X} - \mathbf{A}\mathbf{X} - \mathbf{B}\mathbf{U}_0) \quad (9)$$



## 3. Control Solution

### 3.1 Uncertainty and Disturbance Estimator

#### Standard filter designs:

- Low-pass filters
- $\alpha$  filter
- General linear filter of order  $k$

$$g_{ij} = \frac{\omega}{s + \omega} = \frac{1}{\tau s + 1} \quad (10)$$

$$g_{ij} = \frac{(1 - \alpha)\tau s + 1}{\tau s + 1} \quad (11)$$

$$g_{ij} = \frac{(\tau s + 1)^k - (\tau s)^k}{(\tau s + 1)^k} \quad (12)$$



## 3. Control Solution

### 3.2 Targeted UDE Filtering

#### Motivation:

- Practical limitations on cut-off frequency of low-pass filter
- State derivative information availability for  $\alpha$  filter
- Improve filter design using system error dynamics

#### Error dynamics:

$$\begin{aligned}
 X(s) &= -[sI - (A - BK)]^{-1} \times \\
 &\quad (I - G_f(s))D(s) \\
 X(s) &= Z(s)D(s) \tag{13}
 \end{aligned}$$

#### Disturbance estimation error:

$$\begin{aligned}
 \tilde{D}(s) &= D(s) - \hat{D}(s) \\
 &= (I - G_f(s))D(s) \\
 \tilde{D}(s) &= W(s)D(s) \tag{14}
 \end{aligned}$$



## 3. Control Solution

### 3.2 Targeted UDE Filtering

#### Proposed targeted filter:

- Maximize disturbance attenuation effect at most significant frequency
- Combination of linear filter and bandpass filter

$$g_{ij} = \frac{2\omega_1 s + \omega_1^2}{s^2 + 2\omega_1 s + \omega_1^2} + K \left( \frac{\frac{\omega_2}{Q} \mu + s^2}{s^3 + s^2 + \frac{\omega_2}{Q_2} s + \omega_2^2} \right) \quad (15)$$

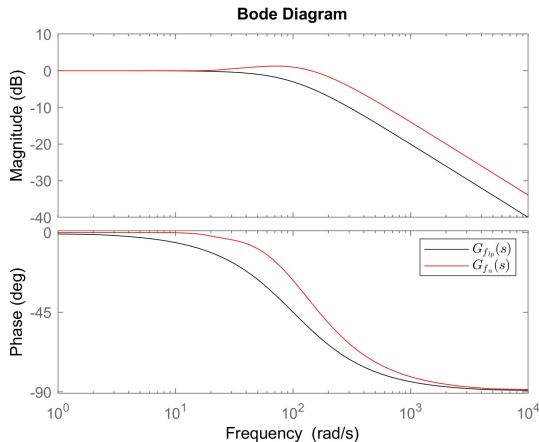
## 3. Control Solution

### 3.2 Targeted UDE Filtering

**Comparison to low-pass filter:**

$G_{f_{lp}}(s)$ , low-pass filter matrix

$G_{f_n}(s)$ , proposed filter matrix

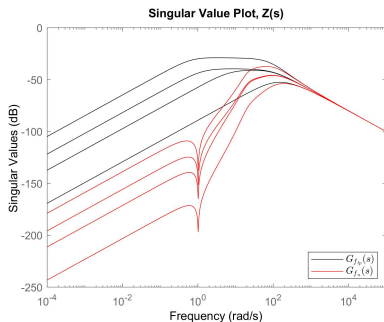


**Fig 4:** Bode plot of low-pass filter ( $\omega = 100$  rad/s, black) frequency response vs proposed filter (red).

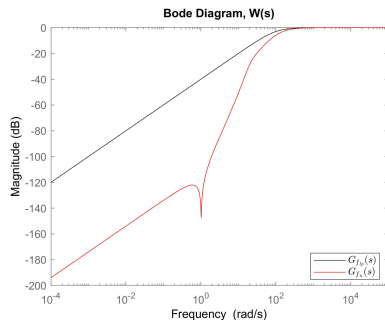
## 3. Control Solution

### 3.2 Targeted UDE Filtering

Error dynamics:



Disturbance Estimation Error:



## 3. Control Solution

### 3.2 Targeted UDE Filtering

#### Frequency identification:

- Frequency of disturbance is required to design filter
- With UDE, disturbances are lumped, combined frequency response required
- Frequency content can be obtained by analyzing spectral density of state signals

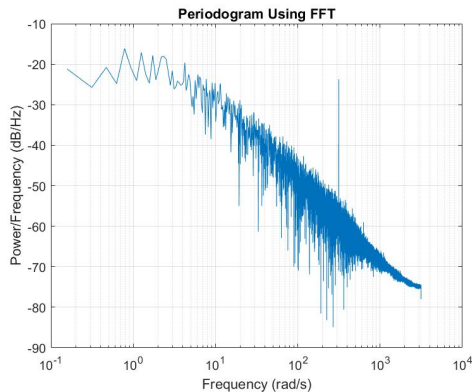


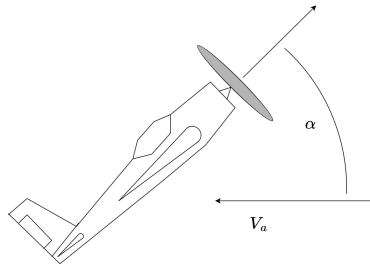
Fig 5: Power spectral density of state variables during harrier aerobatic maneuver.

## 3. Control Solution

### 3.3 Simulation Results

#### Harrier maneuver test case:

- Control law test on harrier maneuver linearized system
- Goal to maintain altitude and pitch angle ( $\theta = 45^\circ$ ),  $\theta = \alpha$  with no wind



# 3. Control Solution

## 3.3 Simulation Results

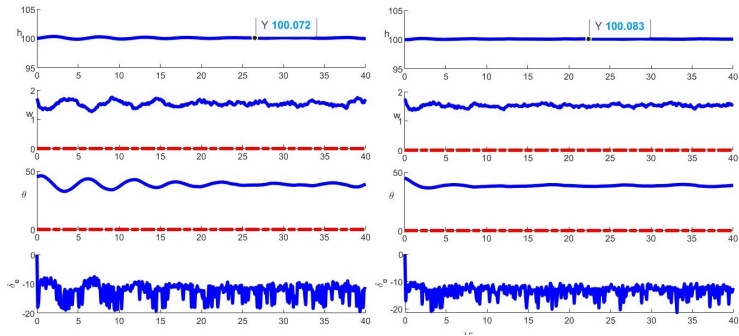


Fig 6: Comparison of altitude tracking problem while maintaining harrier maneuver, with low-pass filter (left) and the novel proposed filter (right).

## Conclusion:

- This paper discusses the use of targeted UDE filtering to improve disturbance rejection capabilities, with application to a fixed-wing UAV system performing a harrier maneuver
- Filter design uses disturbance frequency content and system dynamics



**Thank you!**



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