A Model-Based Drift Correction Control for UAV in GNSS-Degraded Environments

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Outline

1. Introduction

1.1 Motivation

- UAV applications have been increasing in recent years
  - Surveillance
  - Search mission
  - Aerial photography
  - ...
- UAV flight safety has become a critical issue
  - GNSS-degraded/GNSS-denied environment
- A reliable algorithm that can reduce the influence of failures from the UAV operation
  - Design a controller to minimize the tracking error in GNSS-degraded environment

1. Introduction

1.2 Literature Review

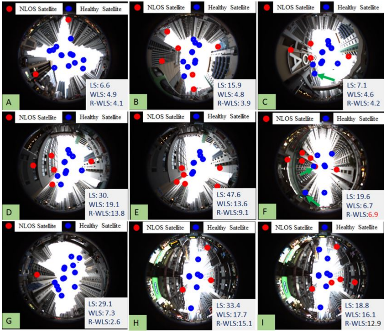

Fig 1: Fish eye camera[1]

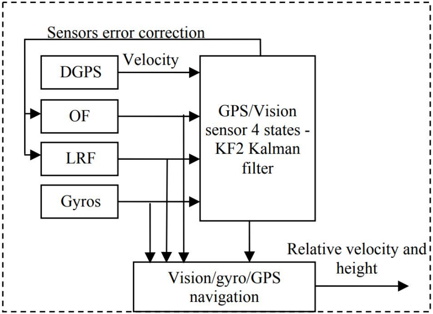

Fig 2: Integration of GPS/vision[2]

[1] Tightly Coupled GNSS/INS Integration via Factor Graph and Aided by Fish-Eye Camera

[2] Integration of GPS/INS/vision sensors to navigate unmanned aerial vehicles

1. Introduction

1.2 Literature Review

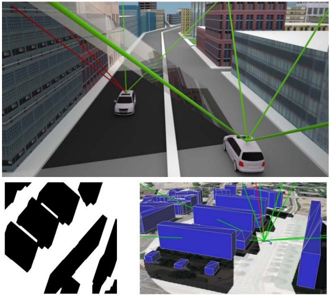

Fig 3: Shadow map[3]

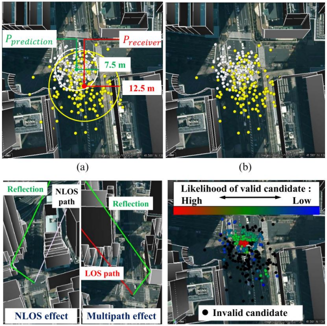

Fig 4: 3-D building map[4]

[3] Evaluation of Shadow Maps for Non-Line-of-Sight Detection in Urban GNSS Vehicle Localization with VANETs—The GAIN Approach

[4] GNSS/Onboard Inertial Sensor Integration With the Aid of 3-D Building Map for Lane-Level Vehicle Self-Localization in Urban Canyon

1. Introduction

1.2 Literature Review

- Kemin Zhou proposed an approach that is based on a variation of all controller parametrization. This Generalized Internal Model Control (GIMC) consists of two parts: a nominal performance controller and a robustness controller.[5]

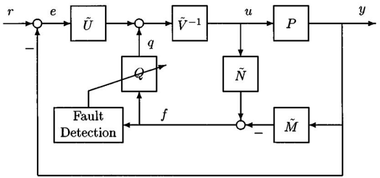

Fig 5: GIMC[5]

- Xiang Chen proposed a new paradigm which renders the exactly same nominal control performance if there is no modeling mismatch for the plant, but provides automatic robust recovery of the nominal performance when the modeling error is present[6]

[5] A New Approach to Robust and Fault Tolerant Control
[6] Revisit of LQG Control–A New Paradigm with Recovered Robustness

2. Problem Formulation

Suppose the plant $P(s)$ is represented by

$$P(s) = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] = C(sI - A)^{-1}B + D \tag{1}$$

which is controlled by a nominal controller $K(s)$ in a standard feedback loop

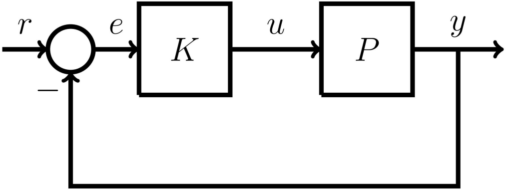

Fig 6: Nominal controller block diagram

2. Problem Formulation

Youla-Kucera Lemma

Let the stabilizing controller $K_0(s) = \tilde{V}(s)^{-1}\tilde{U}(s)$ and the plant $P(s) = \tilde{M}(s)^{-1}\tilde{N}(s)$ be the left coprime factorization.
Then all stabilizing controller $K(s)$ can be expressed as:

$$K(s) = (\tilde{V}(s) - \tilde{Q}(s)\tilde{N}(s))^{-1}(\tilde{U}(s) + \tilde{Q}(s)\tilde{M}(s)) \tag{2}$$

given that $\tilde{Q} \in H_\infty$ such that $det(\tilde{V}(\infty) - \tilde{Q}(\infty)\tilde{N}(\infty)) \neq 1$.

2. Problem Formulation

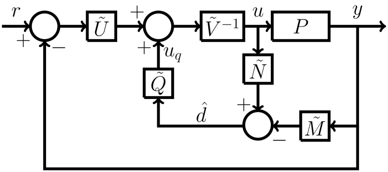

Fig 7: Youla-Kucera parametrization of the nominal controller

With the Youla-Kucera parametrization, the new but equivalent control structure is shown above. Assume that (A, B) is stabilizable, (C, A) is detectable and $A + LC$ is stable. Then one left coprime representation of the plant $P(s) = \tilde{M}(s)^{-1}\tilde{N}(s)$ can be chosen as[7]:

$$\begin{bmatrix} \tilde{N}(s) & \tilde{M}(s) \end{bmatrix} = \left[\begin{array}{c|cc} A+LC & B+LD & L \\ \hline C & D & I \end{array}\right]$$

[7] A New Controller Architecture for High Performance, Robust, and Fault-Tolerant Control

2. Problem Formulation

$$\hat{d} = \tilde{N}(s)u - \tilde{M}(s)y \tag{4}$$

Then the representation becomes:

$$\begin{aligned} \dot{\hat{x}} &= (A + LC)\hat{x} + (B + LD)u - Ly \\ \hat{d} &= C\hat{x} + Du - y \end{aligned} \tag{5}$$

In a GNSS-degraded environment, the output signal of the system is not ground truth anymore. Instead, it is polluted by the GNSS drift, d. The goal of this problem is to design a nominal controller and a robust controller under the GIMC framework such that the true state x converges to the reference signal x_r when the actual sensor output $\bar{y}$ is not equal to the true output y.

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + Bu + L(\hat{y} - \bar{y}) \\ \hat{d} &= \hat{y} - \bar{y} \end{aligned}$$

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2. Problem Formulation

To simplify the overall Youla-Kucera parametrization, we let $K(s) = \tilde{V}^{-1}(s)\tilde{U}(s)$ and $Q(s) = \tilde{V}^{-1}(s)\tilde{Q}(s)$. Then the simplified controller structure based on the Youla-Kucera parametrization is shown below

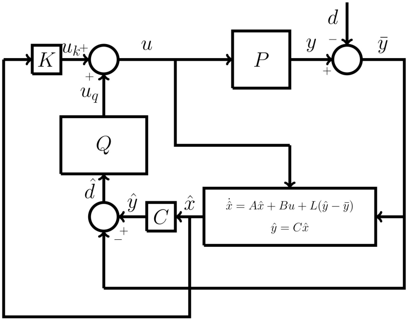

Fig 8: Simplified Youla-Kucera parametrization of the nominal controller

3. Controller Design

Controller Overview

There will be two separate designs of controllers, a nominal controller $K(s)$ and a robust controller $Q(s)$. In this control system design, the nominal controller $K(s)$ is an optimal LQG controller, and the robust drift correction controller $Q(s)$ is a conservative H_∞ controller.

3. Controller Design

3.1 Nominal Controller

The control objective is to find an admissible controller which stabilizes the closed-loop system and minimizes the following cost function:

$$J = \int_0^\infty [x^T Q_k x + u^T R_k u] dt \tag{7}$$

Assume that (A, B) is stabilizable.

$$u^* = -R_k^{-1} B^T P_k x = Kx \tag{8}$$

where

$$J^*(x) = x^T P_k x \tag{9}$$

Then, the optimal solution P_k can be solved by the algebraic Riccati equation:

$$0 = P_k A + A^T P_k - P_k B R_k^{-1} B^T P_k + Q_k \tag{10}$$

3. Controller Design

3.2 Observer

A Luenberger observer is designed to estimate the full states from system input u and sensor output $\bar{y}$. Similarly, assume that (C, A) is detectable.

$$\dot{\hat{x}} = A\hat{x} + Bu + L(\hat{y} - \bar{y}) \tag{11}$$

The error dynamics $e = x - \hat{x}$ simplify to:

$$\dot{e} = (A + LC)e \tag{12}$$

As long as $A + LC$ is Hurwitz, the observer is convergent.
Similar to the linear quadratic controller, the optimal observer gain can be obtained by solving the following algebraic Riccati equation:

$$0 = P_l A^T + AP_l - P_l C^T R_l^{-1} CP_l + Q_l \tag{13}$$

Then the observer gain can be obtained as:

$$L = -P_l C^T R_l^{-1} \tag{14}$$

3. Controller Design

3.3 Robust Controller

On top of the nominal controller, an additional robust controller H_∞ is shown as a Linear Fractional Transformation (LFT).

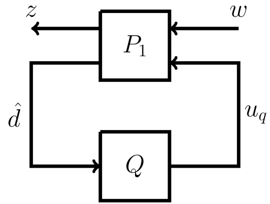

Fig 9: H_∞ control in LFT

In order to minimize the loss of generality, a Linear Matrix Inequality (LMI) method is adopted to solve the control problem.

3. Controller Design

3.3 Robust Controller

The plant $P(s)$ is converted to $P_1(s)$:

$$P_1(s) = \left[\begin{array}{c|cc} A_1 & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right] \tag{15}$$

after implementing the K and L and introducing $e = x - \hat{x}$:

$$\begin{aligned} \dot{x}_q &= A_1 x_q + B_1 w + B_2 u_q \\ z &= C_1 x_q + D_{11} w + D_{12} u_q \\ \hat{d} &= C_2 x_q + D_{21} w + D_{22} u_q \\ u_q &= Q\hat{d} \end{aligned} \tag{16}$$

3. Controller Design

3.3 Robust Controller

The controller defined as $u_q = Q(s)\hat{d}$ where

$$Q(s) = \left[\begin{array}{c|c} A_q & B_q \\ \hline C_q & D_q \end{array}\right] \tag{17}$$

is represented in a 4-matrix state-space form. The goal is to choose $Q(s)$ to minimize

$$||P_{11} + P_{12}(I - QP_{22}^{-1}QP_{21})|| \tag{18}$$

which is equivalently choosing $\left[\begin{array}{c|c} A_q & B_q \\ \hline C_q & D_q \end{array}\right]$ to minimize

$$\left[\begin{array}{c|c} \begin{bmatrix} A_1 & 0 \\ 0 & A_q \end{bmatrix} + \begin{bmatrix} B_2 & 0 \\ 0 & B_q \end{bmatrix} \begin{bmatrix} I & -D_q \\ -D_{22} & I \end{bmatrix}^{-1} \begin{bmatrix} 0 & C_q \\ C_2 & 0 \end{bmatrix} & \begin{bmatrix} B_1 + B_2 D_q S D_{21} \\ B_q S D_{21} \end{bmatrix} \\ \hline \begin{bmatrix} C_1 & 0 \end{bmatrix} \begin{bmatrix} D_{12} & 0 \end{bmatrix} \begin{bmatrix} I & -D_q \\ -D_{22} & I \end{bmatrix}^{-1} \begin{bmatrix} 0 & C_q \\ C_2 & 0 \end{bmatrix} & D_{11} + D_{12} D_q S D_{21} \end{array}\right]_{H_\infty} \tag{19}$$

where $S = (I - D_{22}D_q)^{-1}$. The only assumption made is that $(I - D_{22}D_q)$ is invertible.

3. Controller Design

3.3 Robust Controller

After the transition, it yields the system:

$$\left\| \left[\begin{array}{c|c} \begin{bmatrix} A_1 + B_2 D_{q2} C_2 & B_2 C_{q2} \\ B_{q2} C_2 & A_{q2} \end{bmatrix} & \begin{matrix} B_1 + B_2 D_{q2} D_{21} \\ B_{q2} D_{21} \end{matrix} \\ \hline \begin{bmatrix} C_1 + D_{12} D_{q2} C_2 & D_{12} C_{q2} \end{bmatrix} & D_{11} + D_{12} D_{q2} D_{21} \end{array}\right] \right\|_{H_\infty} \leq \gamma \tag{20}$$

The new parameters A_{q2}, B_{q2}, C_{q2} and D_{q2} are proposed to replace the variables A_q, B_q, C_q and D_q to make the system affine in $\left[\begin{array}{c|c} A_{q2} & B_{q2} \\ \hline C_{q2} & D_{q2} \end{array}\right]$

$$\begin{aligned} A_{q2} &= A_q + B_q S D_{22} C_q \\ B_{q2} &= B_q S \\ C_{q2} &= (I + D_q S D_{22}) C_q \\ D_{q2} &= D_q S \end{aligned} \tag{21}$$

3. Controller Design

3.3 Robust Controller

The controller can be obtained by solving the following equations in order:

$$\begin{aligned} D_q &= (I + D_{q2} D_{22})^{-1} D_{q2} \\ B_q &= B_{q2}(I - D_{22} D_q) \\ C_q &= (I - D_q D_{22}) C_{q2} \\ A_q &= A_{q2} - B_q (I - D_{22} D_q)^{-1} D_{22} C_q \end{aligned} \tag{22}$$

where $\left[\begin{array}{c|c} A_{q2} & B_{q2} \\ \hline C_{q2} & D_{q2} \end{array}\right] = \begin{bmatrix} X_2 & X_1 B_2 \\ 0 & I \end{bmatrix}^{-1} \left[\begin{bmatrix} A_n & B_n \\ C_n & D_n \end{bmatrix} - \begin{bmatrix} X_1 A_1 Y_1 & 0 \\ 0 & 0 \end{bmatrix} \right] \begin{bmatrix} Y_2^T & 0 \\ C_2 Y_1 & I \end{bmatrix}$

4. Simulation

4.1 Quadrotor Model

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \tag{23}$$

$$\mathbf{x} = \begin{bmatrix} x & y & z & u & v & w \end{bmatrix}^T \tag{24}$$

$$\mathbf{y} = \begin{bmatrix} x & y & z \end{bmatrix}^T \tag{25}$$

$$\mathbf{u} = \begin{bmatrix} \theta & \phi & \psi & \boxed{T} \end{bmatrix}^T \tag{26}$$

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & \cos\theta\cos\psi & \cos\psi\sin\theta\sin\phi - \sin\psi\cos\phi & \cos\psi\sin\theta\cos\phi + \sin\psi\sin\phi \\ 0 & 0 & 0 & \cos\theta\sin\psi & \sin\psi\sin\theta\sin\phi + \cos\psi\cos\theta & \cos\phi\sin\psi\sin\theta - \cos\psi\sin\phi \\ 0 & 0 & 0 & -\sin\theta & \sin\phi\cos\theta & \cos\theta\cos\phi \\ 0 & 0 & 0 & 0 & r & -q \\ 0 & 0 & 0 & -r & 0 & p \\ 0 & 0 & 0 & q & -p & 0 \end{bmatrix} \tag{27}$$

$$\mathbf{B} = \begin{bmatrix} \omega\cos\psi\cos\theta\cos\phi & 0 & -v\cos\psi\cos\phi & 0 \\ 0 & -\omega\cos\psi\cos\phi & u\cos\psi\cos\theta & 0 \\ -u\cos\theta & v\cos\phi\cos\theta & 0 & 0 \\ g\cos\psi\cos\theta\cos\phi & 0 & 0 & 0 \\ 0 & -g\cos\psi\cos\phi & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{m} \end{bmatrix} \tag{28}$$

4. Simulation

4.2 Result Discussion

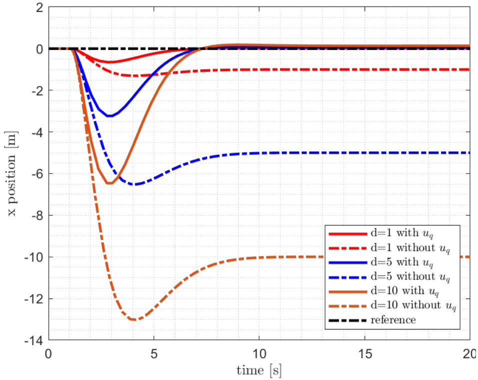

Fig 10: Constant reference

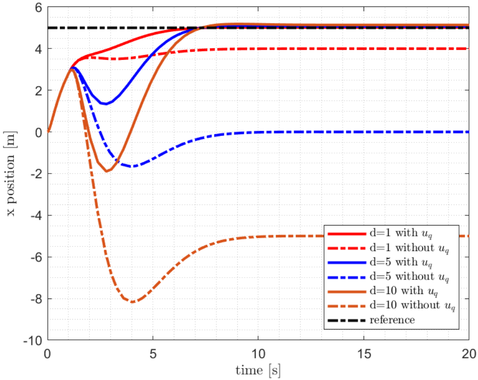

Fig 11: Step reference

4. Simulation

4.2 Result Discussion

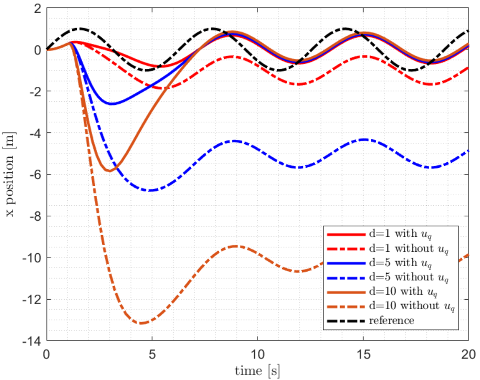

Fig 12: Sinusoidal reference

4. Simulation

4.2 Result Discussion

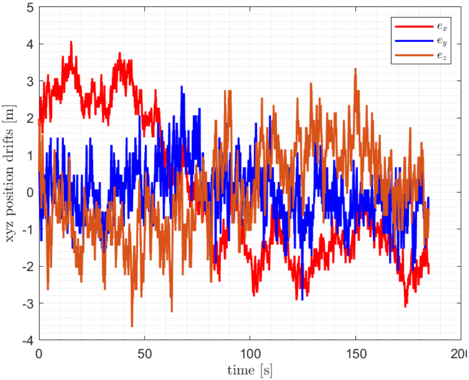

Fig 13: GNSS error

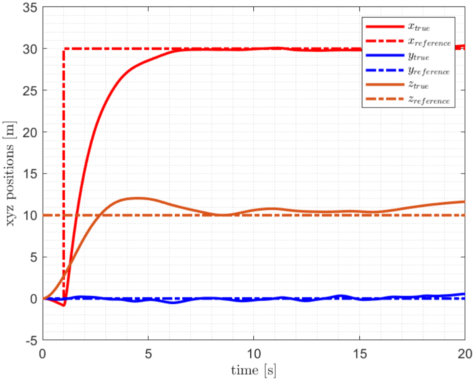

Fig 14: Tracking performance

5. Conclusion

This paper

- Customizes the GIMC control framework to improve tracking performance in a GNSS-degraded environment
- Applies nominal controller, observer, and robust controller to the GIMC control framework to solve GNSS drift problem
- Collects real-world data to justify the effectiveness of the proposed control method in the simulation

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