Institute for Aerospace Studies
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Collision Avoidance Path Planning Between UAS with Tethered Payloads

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Introduction: Motivation

The **New Frontiers Drone Water Sampling Demonstration Project** in collaboration with Queen's University aims to use multiple UAS to conduct autonomous sample collection over various water bodies.

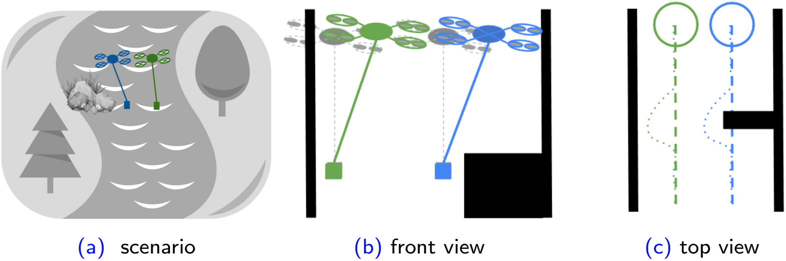
(a) scenario (b) front view (c) top view

Figure 1: water sampling with multiple slung-load drones in an obstacle-laced environment

Introduction: Objectives

Demonstrate collision-free path planning and control for two slung-load multirotors flying in close proximity.

- Find a controller that can follow separate trajectories for the payload and the drone
- Set up the control problem such that the motion is collision-free

Literature Review

- Control of UAS to track a given trajectory while suppressing residual payload motion[1][2][3]
- Control of passively suspended load through UAS actuation using a geometric controller with a cascade structure[4]
- Optimal control to perform tracking or planning and agile maneuver simulations[5][6][7]

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[1] Raffo and Almeida, "Nonlinear robust control of a quadrotor UAV for load transportation with swing improvement".

[2] Qian and Liu, "Path-Following Control of A Quadrotor UAV with A Cable-Suspended Payload under Wind Disturbances".

[3] Pizetta, Brandao, and Sarcinelli-Filho, "Control and Obstacle Avoidance for an UAV Carrying a Load in Forestal Environments".

[4] Sreenath, Michael, and Kumar, "Trajectory generation and control of a quadrotor with a cable-suspended load - A differentially-flat hybrid system".

[5] Crousaz, Farshidian, and Buchli, "Aggressive Optimal Control for Agile Flight with a Slung Load".

[6] Son et al., "Real-time optimal trajectory generation and control of a multi-rotor with a suspended load for obstacle avoidance".

[7] Potdar, Croon, and Alonso-Mora, "Online trajectory planning and control of a MAV payload system in dynamic environments".

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NMPC: Problem Formulation

The discretized optimization problem given as

$$\begin{aligned}
\min_{[\mathbf{u},\mathbf{x}]} \quad & J = c_t\left(\boldsymbol{x}_N\right) + \textstyle\sum_{k=0}^{N-1} c_s\left(\boldsymbol{x}_k, \boldsymbol{u}_k\right) & \\
\text{s.t.} \quad & \boldsymbol{x}_0 = \boldsymbol{x}(t) & \text{(Initial Estimated State)} \\
& \boldsymbol{x}_{k+1} = f\left(\boldsymbol{x}_k, \boldsymbol{u}_k\right) & \text{(Discretised Dynamics)} \\
& g\left(\boldsymbol{x}_k, \boldsymbol{u}_k\right) \geq 0 & \text{(Inequality Constraints)} \\
& \boldsymbol{x}_k \in \mathcal{X} & \text{(State Constraints)} \\
& \boldsymbol{u}_k \in \mathcal{U} & \text{(Input Constraints)}\ .
\end{aligned}$$

NMPC: Problem Formulation

1. Cost function

$$c_y = \|\mathbf{y}_{ref}(k+1) - \mathbf{y}(k+1)\|_{\mathbf{Q}}, \tag{1}$$

$$c_{\Delta u} = \|\mathbf{u}(k) - \mathbf{u}(k-1)\|_{\mathbf{R}} \tag{2}$$

- $\mathbf{y}^\top = \begin{bmatrix} \|\mathbf{r}_L\| & \mathbf{p}_q{}^\top \end{bmatrix}$
- $\mathbf{y}_{ref}$: desired values to be tracked at each stage k
- $\mathbf{Q} \in \mathbb{R}^{n_y \times n_y}$: tuning weight for the navigation cost c_y
- $\mathbf{R} \in \mathbb{R}^{n_u \times n_u}$: tuning weight for the input move suppression cost $c_{\Delta u}$

NMPC: Problem Formulation

2 Slung-load UAS dynamics

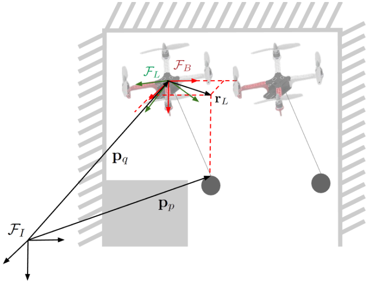


Figure 2: Frame definition and slung-load UAS geometry

$$\Sigma_{\text{rot}} : \begin{cases} \mathbf{J}\dot{\omega}_b = -\omega_b^{\times}\mathbf{J}\boldsymbol{\omega}_b + \tau \\ \dot{\mathbf{R}}_{ib} = \mathbf{R}_{ib}\boldsymbol{\omega}_b^{\times} \\ \mathbf{f}_L = -f\mathbf{R}_{IB}\mathbf{1}_3 \end{cases}$$

$$\Sigma_{\text{trans}} : \begin{bmatrix} \dot{\boldsymbol{p}} \\ \dot{\boldsymbol{v}} \end{bmatrix} = \begin{bmatrix} \mathbf{0}_{5\times5} & \mathbf{1}_{5\times5} \\ \mathbf{0}_{5\times5} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix} \begin{bmatrix} \boldsymbol{p} \\ \boldsymbol{v} \end{bmatrix} + \begin{bmatrix} \mathbf{0}_{5\times1} \\ \mathbf{M}^{-1}\mathbf{G} \end{bmatrix} + \begin{bmatrix} \mathbf{0}_{5\times5} \\ \mathbf{M}^{-1}\mathbf{F} \end{bmatrix}$$

$$\mathbf{x}_{\text{trans}}^{\top} = \left[\mathbf{r}_L{}^{\top}\ \mathbf{p}_q{}^{\top}\ \mathbf{v}_L{}^{\top}\ \mathbf{v}_q{}^{\top}\right] = \left[\boldsymbol{p}^{\top}\ \boldsymbol{v}^{\top}\right]$$

$$\mathbf{x}_{\text{rot}} = \{\boldsymbol{\omega}_B,\ \mathbf{R}_{IB}\}$$

$$\mathbf{u} = \{f,\ \mathbf{R}_{IB}\}$$

NMPC: Problem Formulation

3 Constraints

- Workspace and input limits

$$\mathcal{W}_{min} \leq \mathbf{p}_q,\ \mathbf{p}_p \leq \mathcal{W}_{max}, \tag{3}$$

$$\mathcal{U}_{min} \leq \mathbf{u} \leq \mathcal{U}_{max}. \tag{4}$$

- Obstacle distance

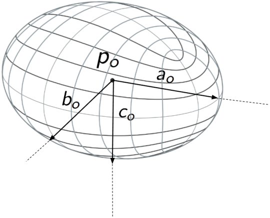


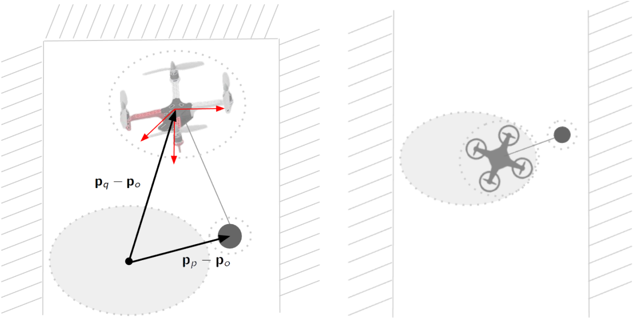
Figure 3: Ellipsoidal obstacle with fixed position $\mathbf{p}_o$ and semi-principal axes (a_o, b_o, c_o)

$$d_o(\mathbf{p}, \mathbf{p}_o) > 0, \tag{5}$$

where $d_o(\mathbf{p}, \mathbf{p}_o) = \|\mathbf{p} - \mathbf{p}_o\|_\Omega - 1$ and $\Omega = \text{diag}\left(1/a_o^2, 1/b_o^2, 1/c_o^2\right)$.

NMPC: Plant and Obstacle Setup

- A slung-load drone attempting to pass through a corridor while avoiding a low-sitting obstacle



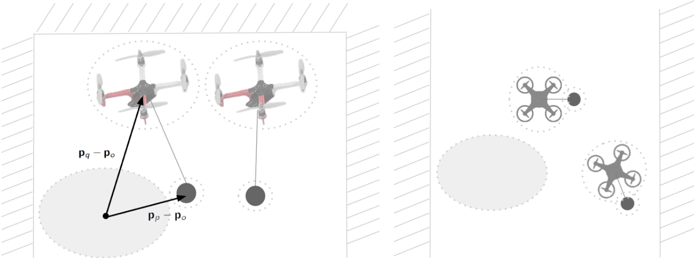
(a) front view (b) top view

Figure 4: Slung-load drone navigating corridor

NMPC: Plant and Obstacle Setup

- Default setup with two slung-load systems and referring to all rigid bodies as obstacles

$$\mathbf{x} = \{\mathbf{x}_{\text{trans }1},\ \mathbf{x}_{\text{rot }1}, \mathbf{x}_{\text{trans }2},\ \mathbf{x}_{\text{rot }2}\},\quad \mathbf{u}^{\top} = \left[f_1^{\top}\ \boldsymbol{\tau}_1^{\top}\ f_2^{\top}\ \boldsymbol{\tau}_2^{\top}\right]$$



(a) front view

(b) top view

Figure 5: Two full slung-load systems

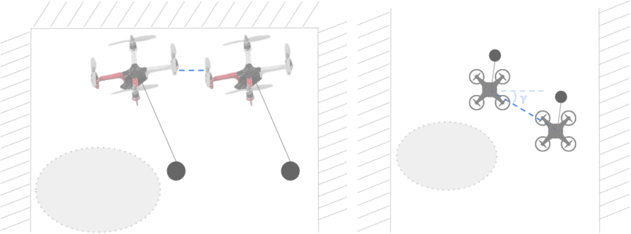
NMPC: Plant and Obstacle Setup

- Coordinated planning with two slung-load systems

$$\mathbf{x} = \{\mathbf{x}_{\text{trans}}, \mathbf{x}_{\text{rot}}\}, \quad \mathbf{u}^\top = \{f, \mathbf{R}_{IB}, \gamma\}$$

- Independent tracking with two slung-load systems

$$\mathbf{y}^\top = \begin{bmatrix}\mathbf{r}_L^\top & \mathbf{p}_q^\top\end{bmatrix}$$



(a) front view (b) top view

Figure 6: Two slung-load systems orbiting around each other

NMPC: MATLAB Setup

- Default simulation parameters

$$\mathrm{m_{pld}} = 1kg,\ \mathrm{m_{dr}} = 1kg,\ \text{Cable length} = 1m,\ \mathbf{x}_{q0}^{\top} = [0\ 0\ 1.5]$$

$$\text{Simulation area (W x L x H): } 2 \times 10 \times 9$$

$$p = 20,\ T_s = 0.01s, \mathbf{y} = \begin{bmatrix} \|\mathbf{r}_L\| \\ \mathbf{p}_q \end{bmatrix}$$

- Coordinated drone-pair simulations

$$p = 20 \text{ or } 60$$

Planning stage:

$$\underbrace{\begin{bmatrix} \boldsymbol{v} \\ \dot{\boldsymbol{v}} \\ \gamma \end{bmatrix}}_{\dot{\mathbf{x}}} = \begin{bmatrix} \mathbf{0}_{5\times5} & \mathbf{1}_{5\times5} & 0 \\ \mathbf{0}_{5\times5} & -\mathbf{M}^{-1}\mathbf{C} & 0 \\ \mathbf{0}_{5\times5} & \mathbf{0}_{5\times5} & 0 \end{bmatrix} \underbrace{\begin{bmatrix} \boldsymbol{p} \\ \boldsymbol{v} \\ 0 \end{bmatrix}}_{\mathbf{x}} + \begin{bmatrix} \mathbf{0}_{5\times1} \\ \mathbf{M}^{-1}\mathbf{G} \\ 0 \end{bmatrix} + \underbrace{\begin{bmatrix} \mathbf{0}_{5\times5} \\ \mathbf{M}^{-1}\mathbf{F} \\ \gamma \end{bmatrix}}_{\mathbf{u}}$$

Tracking stage:

$$\mathbf{y} = \begin{bmatrix} \mathbf{r}_L \\ \mathbf{p}_q \end{bmatrix}$$

NMPC: MATLAB Setup

- Default simulation parameters

$$\mathrm{m}_{\mathrm{pld}} = 1kg,\ \mathrm{m}_{\mathrm{dr}} = 1kg,\ \text{Cable length} = 1m,\ \mathbf{x}_{q0}^{\top} = [0\ 0\ 1.5]$$

Simulation area (W x L x H): $2 \times 10 \times 9$

$$p = 20,\ T_s = 0.01s, \mathbf{y} = \begin{bmatrix} \|\mathbf{r}_L\| \\ \mathbf{p}_q \end{bmatrix}$$

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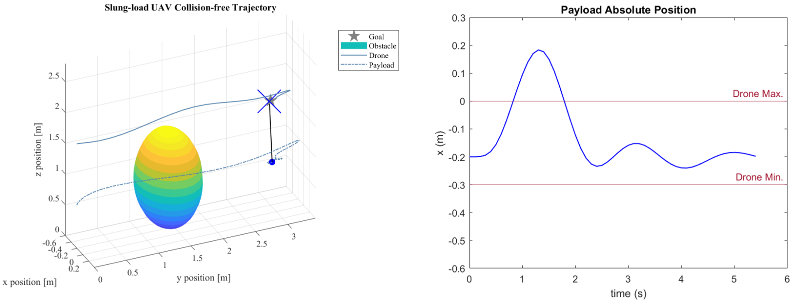
Tracking stage:

$$\mathbf{y} = \begin{bmatrix} \mathbf{r}_L \\ \mathbf{p}_q \end{bmatrix}$$

Results: Obstacle Avoidance

1. Slung-load UAS is flown to $\mathbf{x}_{q_{goal}}^{\top} = [3\ 0\ 1.5]$ avoiding an obstacle



(a) Goal point reached

(b) Payload position

Figure 7: Trajectory of a slung-load UAS passing through a corridor with obstacle

Results: UAV-pair Obstacle Avoidance

2. Attempt to fly two independent slung-load UAS models to $\mathbf{x}_{q_{goal}}^{\top} = [3\ 0\ 5]$ while avoiding a wide obstacle

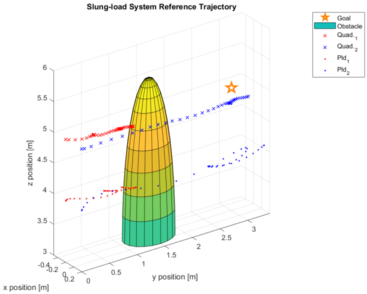


Figure 8: Trajectories of a pair of slung-load UAS avoiding an obstacle

- Unsuccessful mission
- Very long simulation time

Results: UAV-pair Obstacle Avoidance

3. Trajectory planning and tracking for a pair of slung-load UAS maneuvering around each other while avoiding an obstacle

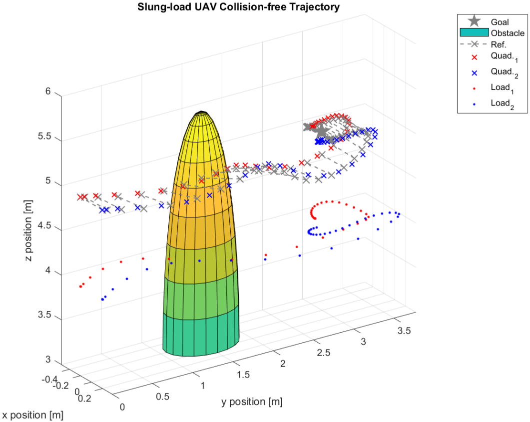
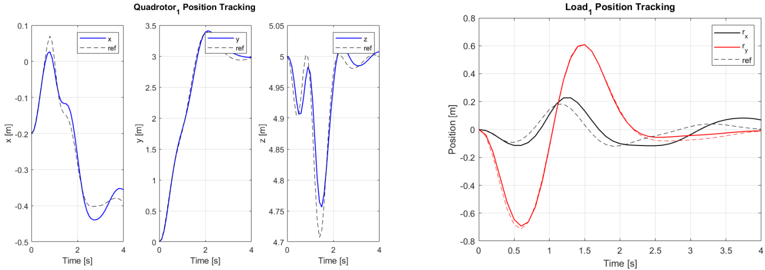


Figure 9: Trajectories of a pair of slung-load UAS for a goal navigation task following the coordinated planning approach

Results: UAV-pair Obstacle Avoidance



(a) Quadrotor trajectory

(b) Payload trajectory

Figure 10: Tracking the reference trajectories for one of the slung-load systems

Conclusion

Research Objective

Demonstrate collision-free path planning and control for two slung-load multirotors flying in close proximity.

- Find a controller that can follow separate trajectories for the payload and the drone
- Set up the control problem such that the motion is collision-free

Next Steps

- Establish a relationship between the required control inputs and the orientation angle to progress toward unifying planning and control
- Experimental validation

Appendix: Slung-load UAS Dynamics

$$\mathbf{v} = \begin{bmatrix} \boldsymbol{v}_q^\top & \boldsymbol{v}_L^\top \end{bmatrix}^\top$$

$$\mathbf{p} = \begin{bmatrix} \boldsymbol{x}_q^\top & \boldsymbol{r}_L^\top \end{bmatrix}^\top$$

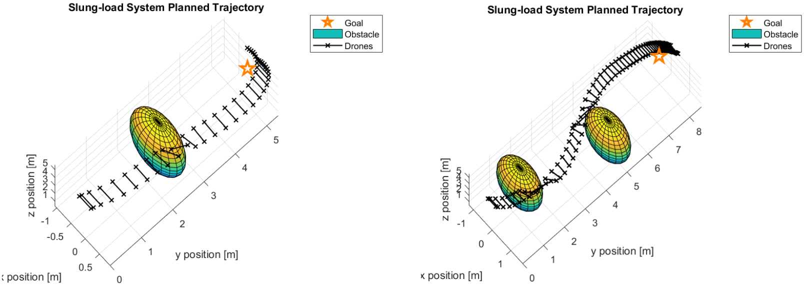
$$\mathbf{G} = -g \begin{bmatrix} (m_p + m_q)\mathbf{1}_3^\top \\ m_p \mathbf{B}\mathbf{1}_3^\top \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 0 & m_p \dot{\mathbf{B}} \\ 0 & m_p \mathbf{B}^\top \dot{\mathbf{B}} \end{bmatrix}$$

$$\mathbf{M} = \begin{bmatrix} (m_p + m_q)\,\mathbf{1} & m_p \mathbf{B} \\ m_p \mathbf{B}^\top & m_p \mathbf{B}^\top \mathbf{B} \end{bmatrix}$$

Appendix: UAV-pair Obstacle Avoidance

- Trajectory planning for a pair of slung-load UAS maneuvering around each other while avoiding wide obstacles



(a)

(b)

Figure 11: Reduced state setup of UAV pair avoiding obstacles in narrow passageway