# Robust Stabilization Of A Quadrotor as A 5G Access Point Using Acceleration Feedback

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# Background and motivation

- Quadrotor drones are widely used in industries such as surveillance, package delivery, map building.
- Quadrotor can act as a mobile 5G access point.
- Usage: search and rescue missions after nature disasters, establish communication in rural areas.
- Key challenge: to maintain the quadrotor as a fixed location under various disturbances such as wind gust.

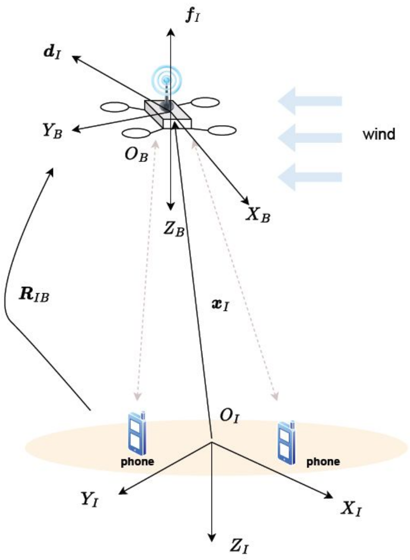



Fig.1 A quadrotor as 5G access point

# Literature survey and related works

- **Adaptive control**
  - The lumped uncertainties are treated as unknown parameters
  - Estimated online by the adaptive control law [1], [2].
  - Better transient performance L1 performance [3].

- **Uncertainty and disturbance estimator(UDE)**
  - A low-pass filtered result of the difference between the force and the acceleration [4].
  - This property ensures that during the transient phase, the system states do not deviate from the desired path too much, avoiding introducing the effect of unmodeled dynamics or singularities.

[1] A. Roberts and A. Tayebi, “Adaptive position tracking of VTOL UAVs,” IEEE Transactions on Robotics, vol. 27, no. 1, pp. 129–142, 2011.
[2] D. Cabecinhas, R. Cunha, and C. Silvestre, “A globally stabilizing path following controller for rotorcraft with wind disturbance rejection,” IEEE Transactions on Control Systems Technology, vol. 23, no. 2, pp. 708–714, 2015.
[3] A. Gahlawat, P. Zhao, A. Patterson, N. Hovakimyan, and E. Theodorou, “L1-GP: L1 Adaptive Control with Bayesian Learning,” in Proceedings of the 2nd Conference on Learning for Dynamics and Control, ser. Proceedings of Machine Learning Research, A. M. Bayen, A. Jadbabaie, G. Pappas, P. A. Parrilo, B. Recht, C. Tomlin, and M. Zeilinger, Eds., vol. 120. PMLR, 2020, pp. 826–837.
[4] L. Qian and H. H. T. Liu, “Robust Control Study for Tethered Payload Transportation Using Multiple Quadrotors,” vol. 45, no. 3, pp. 434–452, jan 2022. [Online]. Available: https://doi.org/10.2514/1.G006173

# Our contribution: UAV + 5G

1) A feedback position stabilization controller is proposed.

2) A Kalman filter is used to provide velocity feedback.

3) A UDE base robust controller to stabilize the quadrotor drone. The UDE utilizes the dynamic model and IMU feedback to estimate and compensate for the disturbances.

# Dynamic Modelling and Problem Formulation

- Rigidbody dynamics:

$$\Sigma_T : \begin{cases} m\dot{\boldsymbol{v}}_I = \boldsymbol{f}_I + \boldsymbol{d} + m\boldsymbol{g}_I \\ \dot{\boldsymbol{x}}_I = \boldsymbol{v}_I \end{cases} ;$$ The position channel

$$\Sigma_R : \begin{cases} \boldsymbol{J}\dot{\boldsymbol{\omega}}_B + \boldsymbol{\omega}_B^{\times}\boldsymbol{J}\boldsymbol{\omega}_B = \boldsymbol{\tau}_B \\ \dot{\boldsymbol{R}}_{IB} = \boldsymbol{R}_{IB}\boldsymbol{\omega}_B^{\times} \end{cases} .$$ The attitude channel

$$\boldsymbol{f}_I = -f\boldsymbol{R}_{IB}\boldsymbol{e}_3$$ The lift force

The flight control problem is the formulated as:

- Given a targeted position, *x*d design a control law fI such that xI -> xd as t->inf

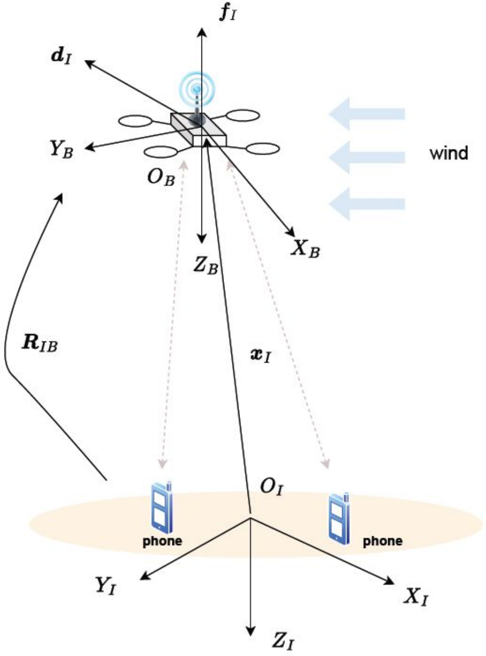



Fig.2 Dynamic modelling

# The Sensor Model

- Assumption: noise in the attitude channel is handled by the attitude motion observer.
- Zero-order hold (ZOH) of continuous states.

$$y_k = y(t_k) + n_k$$

- IMU feedback: provide acceleration
- GPS feedback: provide raw position measurement

ZOH introduce measurement delay: $\Delta T$

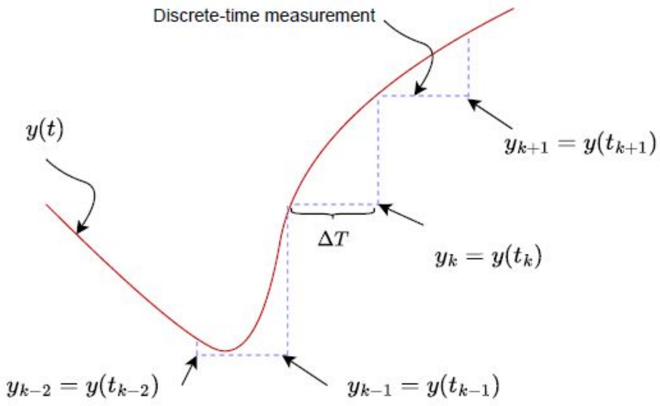



Fig.3 ZOH sampling

# The Kalman Filter

- The position channel is modelled with a discrete-time system.
- Linear Kalman filter is used to combine the GPS measurement with the IMU measurement to produce

$$x_{k+1} = Ax_k + Bu_k + v_k$$
$$y_k = Cx_k + n_k$$

$$A = \begin{bmatrix} 1_{3\times3} & \Delta T \cdot 1_{3\times3} \\ 0_{3\times3} & 1_{3\times3} \end{bmatrix}, B = \begin{bmatrix} 0.5(\Delta T)^2 \cdot 1_{3\times3} \\ \Delta T \cdot 1_{3\times3} \end{bmatrix}$$
$$C = \begin{bmatrix} 1_{3\times3} & 0_{3\times3} \end{bmatrix}.$$

$$v_k \sim \mathcal{N}(0, \eta),\ n_k \sim \mathcal{N}(0, \Sigma).$$

The IMU measurement: $u_k = \dot{v_I}(t_k) + b_k.$

UTIAS

The process noise: $Q = B\zeta B^T. \Rightarrow v_k \sim \mathcal{N}(0, \eta).$

The observation noise: $R = \Sigma.$

Prediction step:

$$x_{k|k-1} = Ax_{k-1|k-1} + Bu_k;$$
$$P_{k|k-1} = AP_{k-1|k-1}A^T + Q.$$

Correction step:

$$\bar{y}_k = y_k - Cx_{k|k-1};$$
$$S_k = CP_{k|k-1}C^T + R;$$
$$K_k = P_{k|k-1}C^T S_k^{-1};$$
$$x_{k|k} = x_{k|k-1} + K_k\bar{y}_k;$$
$$P_{k|k} = (1 - K_kC)P_{k|k-1}.$$

# The Position Stabilization Control Law

- The position stabilization control consists a PD term and the UDE term.
- The total desired force is then converted into a throttle.

The position control law:

$$\boldsymbol{f}_d = -k_v\Big(\boldsymbol{v}_I + k_p h(\boldsymbol{e})\Big) - \hat{\boldsymbol{d}}.$$

Velocity feedback — Position feedback — UDE

$$h(\boldsymbol{x}) := \frac{d\pi(\boldsymbol{x})}{d\boldsymbol{x}} = \boldsymbol{x}/\sqrt{c + \boldsymbol{x}^T\boldsymbol{x}}$$

The attitude tracking law

$$\begin{cases} f = \|\boldsymbol{f}_d\|, \boldsymbol{n}_z = -\boldsymbol{f}_d/f. \\ \boldsymbol{n}_x = \begin{bmatrix}\cos\psi & \sin\psi & -(\cos\psi n_{z,1} + \sin\psi n_{z,2})/n_{z,3}\end{bmatrix}^T \\ \boldsymbol{n}_y = \boldsymbol{n}_z^\times \boldsymbol{n}_x/\|\boldsymbol{n}_z^\times \boldsymbol{n}_x\|, \boldsymbol{R}_{IB,d} = \begin{bmatrix}\boldsymbol{n}_x/\|\boldsymbol{n}_x\| & \boldsymbol{n}_y & \boldsymbol{n}_z\end{bmatrix} \\ \boldsymbol{\tau}_B = -k_\omega\tilde{\boldsymbol{\omega}} - k_R\boldsymbol{e}_R - \tilde{\boldsymbol{\omega}}^\times \boldsymbol{J}\tilde{\boldsymbol{\omega}} + \boldsymbol{\omega}^\times \boldsymbol{J}\boldsymbol{\omega} \\ \quad + \boldsymbol{J}(\tilde{\boldsymbol{\omega}}^\times \tilde{\boldsymbol{R}}^T\boldsymbol{\omega}_d - \tilde{\boldsymbol{R}}^T\dot{\boldsymbol{\omega}}_d) \end{cases}$$

Desired torque

# The Uncertainty and Disturbance Estimator (UDE)

- The key component for achieving stabilization under gust is to use the UDE
- The UDE compares the difference between the measured acceleration and the total known force, i.e. lift force and gravity

Disturbance as a difference between the acceleration and the external force:

$$d = m\dot{v}_I - f_I - mg_I.$$

Disturbance estimation error: $\tilde{d} = \hat{d} - d$

Estimation error dynamics: $\dot{\tilde{d}} = -\lambda \tilde{d}$

Final form of the UDE:

$$\dot{\tilde{d}} = \dot{\hat{d}} - \dot{d} \approx \dot{\hat{d}}$$

$$\dot{\hat{d}} \approx -\lambda(\hat{d} - d)$$

- Assumption: slowly varying disturbance: $\dot{d} \approx 0.$

$$\dot{\hat{d}} \approx -\lambda(\hat{d} - m\dot{v}_I + f_I + mg_I)$$

# Results for Position Stabilization

- A point stabilization mission is used as the test case.

Table 1. System parameters

| Name | Value |
|---|---|
| $m_q$,$kg$ | 2 |
| $J$,$kg \cdot m^2$ | $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} \times 10^{-2}$ |
| $k_v$ | 15 |
| $k_p$ | 1 |
| $c$ | 1 |
| $\lambda$ | 1 |
| $k_\omega$ | 15 |
| $k_R$ | 15 |
| $\Delta T$,$s$ | 0.01 |
| $\zeta$, $m/s^2$ | 0.1 |
| $\sigma$, $m$ | 1 |

Disturbance:

$$d_I = \left( \begin{bmatrix} 6 \\ 7 \\ 0 \end{bmatrix} + \begin{bmatrix} 5sin(0.3t) \\ 2sin(t) \\ 4siin(3t) \end{bmatrix} \right) (N).$$

Initial condition: $x_I = [0, 0, -1]^T (m).$

Target condition: $x_d = [10, 5, -6]^T (m).$

(NED frame)

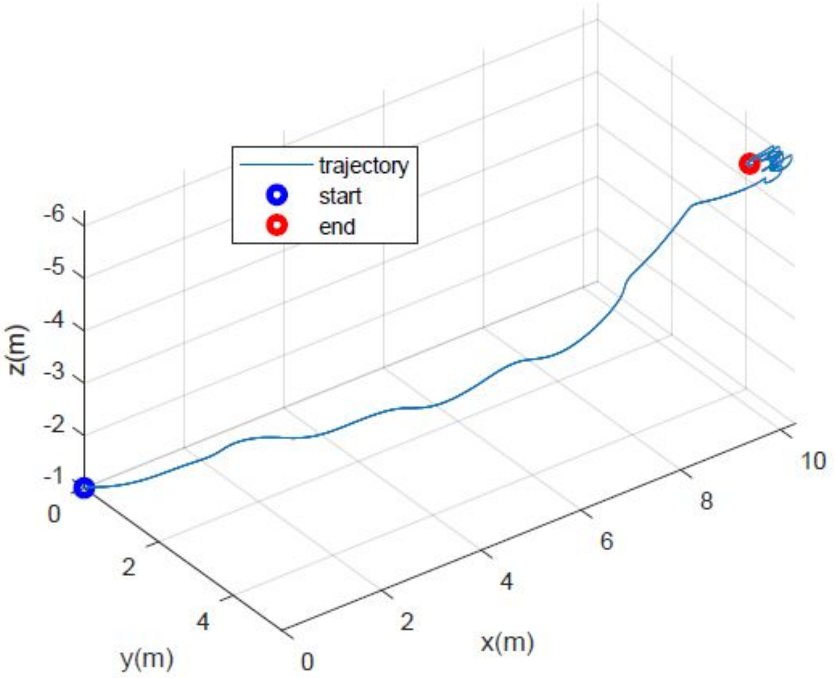



Fig.4 Trajectory of The Quadrotor

# Raw Measurement and Kalman Filter Results

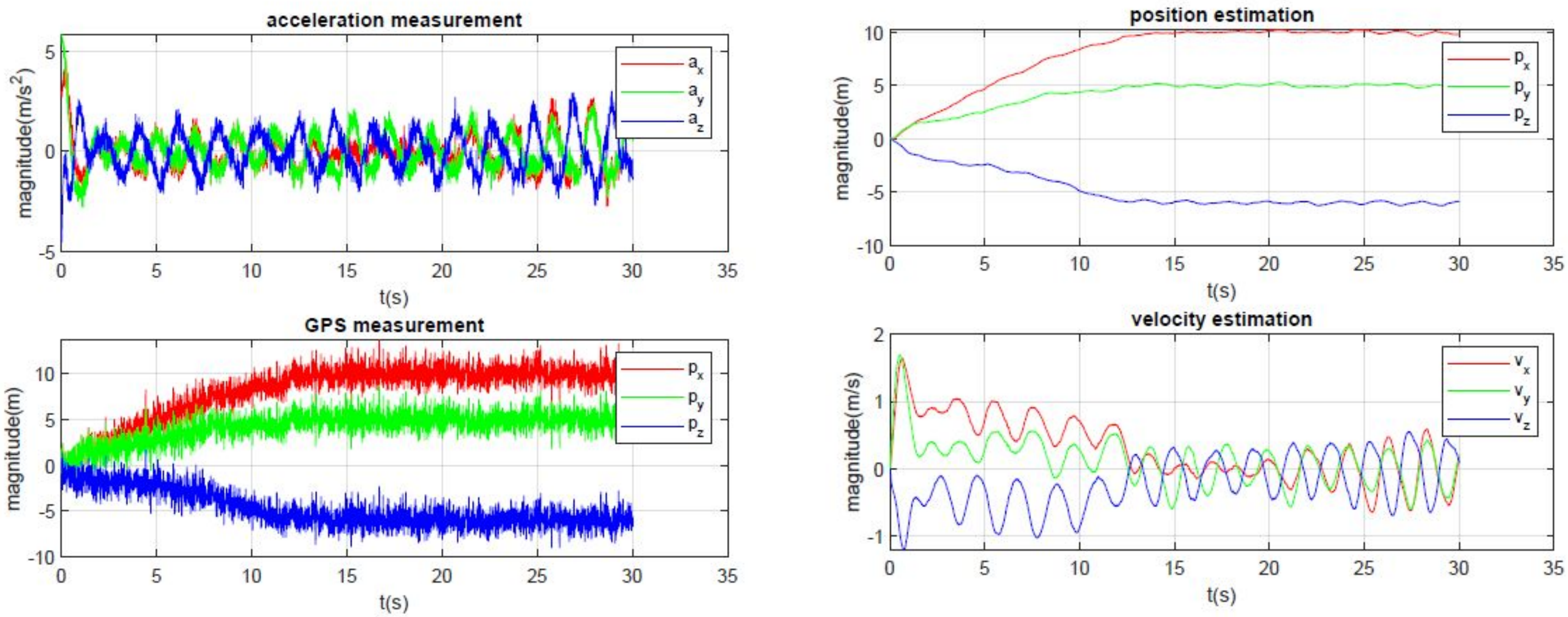



Fig.5 Raw and filtered quadrotor states

# Position Stabilization Error

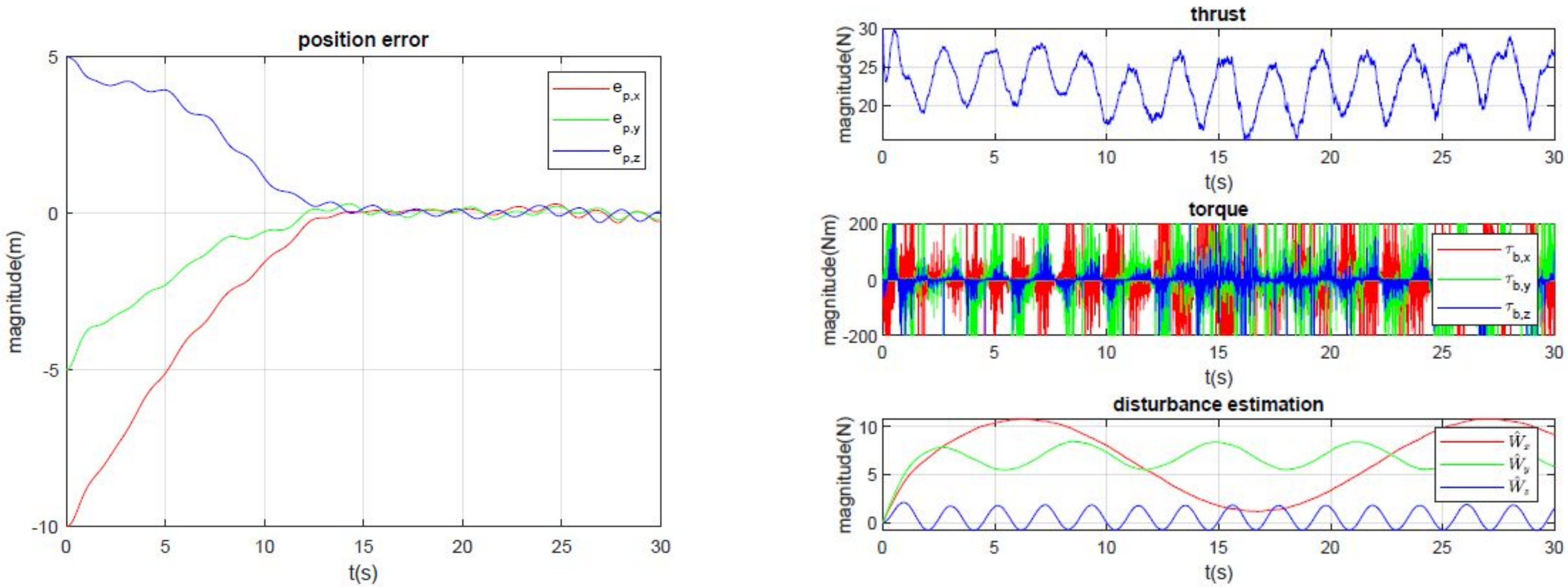



Fig.6 Position stabilization error and estimated disturbances

# A Comparative Study

- A comparative study is done for position stabilization with and without UDE.
- The position error is significantly lower with UDE
- UDE is effective to compensate for constant and time-varying disturbance.

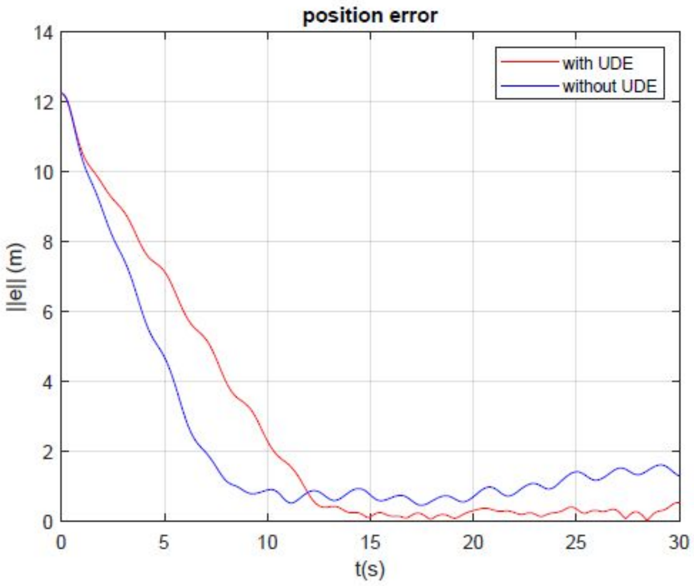
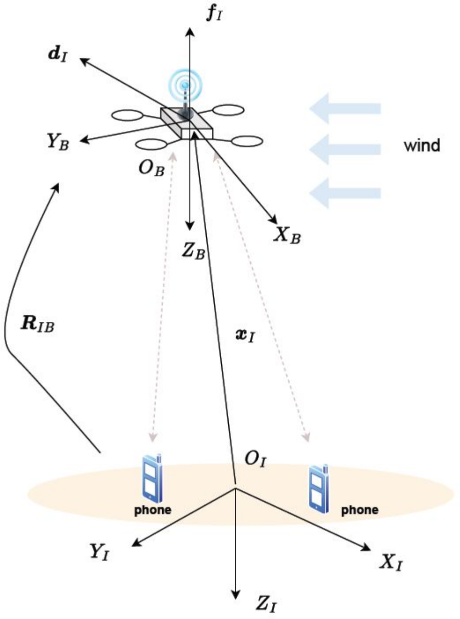


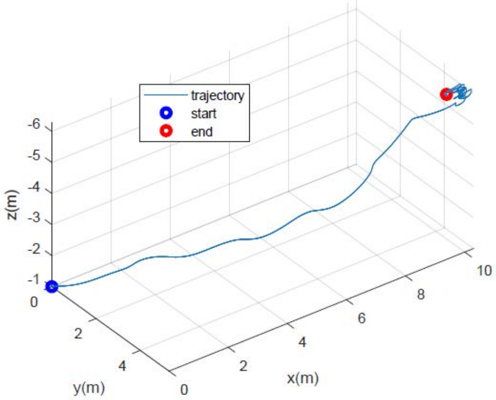

Fig.7 A comparison of results with and without UDE

# Conclusion

## Thank you!

- A flight control design is proposed to utilize a quadrotor drone as a 5G access point.
- A Kalman filter is implemented to obtain the filtered position and velocity.
- A disturbance estimation control law is designed to compensate for the control delay and the gust.





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